7 Pacing Guide

Chapter 7: Right Triangles and Trigonometry

Chapter Table of Contents

PACING GUIDES

Regular Schedule (50-minute classes)

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
Lesson 7.1	Lesson 7.1 (cont.)	Lesson 7.2 Quiz	Investigating Geometry Activity Lesson 7.3	Lesson 7.3 (cont.)
DAY 6	**DAY 7**	**DAY 8**	**DAY 9**	**DAY 10**
Lesson 7.4	Lesson 7.4 (cont.) Mixed Review of Problem Solving	Quiz Lesson 7.5	Lesson 7.6	Lesson 7.6 (cont.)
DAY 11	**DAY 12**	**DAY 13**	**DAY 14**	**DAY 15**
Lesson 7.7	Lesson 7.7 (cont.) Extension 7.7	Quiz Mixed Review of Problem Solving Chapter Review	Chapter Test	Standardized Test Preparation and Practice

Block Schedule (90-minute classes)

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
Lesson 7.1	Lesson 7.2 Quiz Investigating Geometry Activity Lesson 7.3	Lesson 7.3 (cont.) Lesson 7.4	Lesson 7.4 (cont.) Mixed Review of Problem Solving Quiz Lesson 7.5	Lesson 7.6
DAY 6	**DAY 7**	**DAY 8**		
Lesson 7.7 Extension 7.7	Quiz Mixed Review of Problem Solving Chapter Review	Chapter Test Standardized Test Preparation and Practice		

RESOURCE OPTIONS

Chapter/Lesson Resources

Chapter Resource Book
- Parents as Partners
- Teaching Guide/Lesson Plan
- Activity Masters
- Practice (3 levels)
- Study Guide
- Quick Catch-Up for Absent Students
- Problem Solving/Application
- Challenge Practice
- Chapter Review Games and Activities
- Project with Rubric
- Cumulative Review

Notetaking Guide
- Student Workbook and Teacher's Edition

Practice Workbook

Worked-Out Solution Key

Chapter Transparencies, Online only
- Warm-Up Exercises/Daily Homework Quiz
- Notetaking Guide Transparencies
- Homework Answer Transparencies

Teacher Tools Transparencies

Assessment

Assessment Book
- Quizzes
- Chapter Tests (3 levels)
- Standardized and SAT/ACT Chapter Tests
- Alternative Assessments
- Cumulative Tests

Benchmark Tests
- Benchmark Tests, correlated to Remediation Book
- Pre-Course, Mid-Year, and End-of-Year Tests
- Chapter Tests

Spanish Assessment Book

Differentiated Instruction

Differentiated Instruction Resources
- Strategies for Reading Mathematics
- Differentiated Instruction Lesson Notes
- English Learner Lesson Notes
- Inclusion Lesson Notes
- Teaching Strategies with Sample Worksheets
- Tips for New Teachers/Math Background Notes
- Teacher Survival Activities/Bulletin Board Ideas

Student Resources in Spanish

Spanish Study Guide

Remediation Book

Skills Readiness (available online)
- Diagnostic Assessment
- Skill Instruction and Alternative Teaching Strategies
- Skill Practice and Enrichment Masters

Pre-AP Resources
- Pacing and Assignment Guide
- Best Practices
- Copymasters

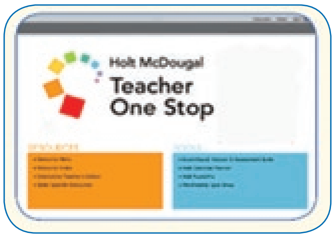
Technology Resources

Plan	***Lesson Plans (available online)***
Teach	***Activity Generator*** ***Power Presentations*** ***Animated Geometry***
Assess	***ExamView™ Assessment Suite*** ***Online Assessment***
Reteach	***@HomeTutor***
Online Resources	***my.hrw.com*** ***eEdition***

Technology Highlights for Each Lesson

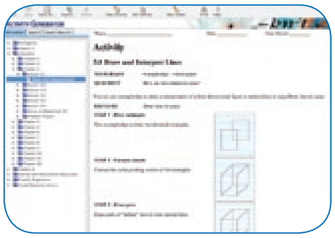
Teacher One-Stop

Easy access to chapter resources and assessments. Includes lesson planning, test generation, and puzzle creation software.



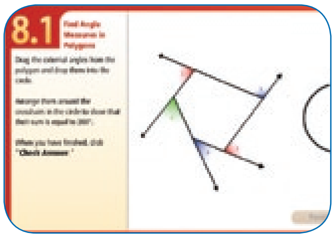
Activity Generator

Leveled, editable activities allow all students to explore a lesson's concepts. Includes teacher notes and closure questions.



Animated Geometry

Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.



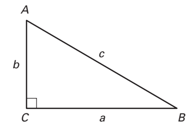
LESSON 7.1

Practice B

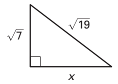
For use with the lesson "Apply the Pythagorean Theorem"

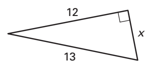
Use △*ABC* to determine if the equation is *true* or *false*.

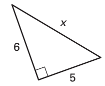
1. $b^2 + a^2 = c^2$
2. $c^2 - a^2 = b^2$
3. $b^2 - c^2 = a^2$
4. $c^2 = a^2 - b^2$
5. $c^2 = b^2 + a^2$
6. $a^2 = c^2 - b^2$

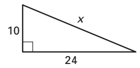


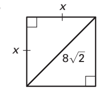
Find the unknown side length. Simplify answers that are radicals. Tell whether the side lengths form a Pythagorean triple.

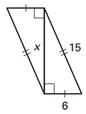
7. 

8. 

9. 

10. 

11. 

12. 

The given lengths are two sides of a right triangle. All three side lengths of the triangle are integers and together form a Pythagorean triple. Find the length of the third side and tell whether it is a leg or the hypotenuse.

13. 40 and 41
14. 12 and 35
15. 63 and 65
16. 28 and 45
17. 56 and 65
18. 20 and 29
19. 80 and 89
20. 48 and 55
21. 65 and 72

Find the area of a right triangle with given leg ℓ and hypotenuse *h*. Round decimal answers to the nearest tenth.

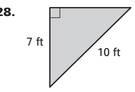
22. $\ell = 8$ m, $h = 16$ m
23. $\ell = 9$ yd, $h = 12$ yd
24. $\ell = 3.5$ ft, $h = 9$ ft
25. $\ell = 9$ mi, $h = 10$ mi
26. $\ell = 21$ in., $h = 29$ in.
27. $\ell = 13$ cm, $h = 17$ cm

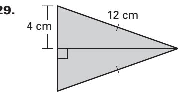
LESSON 7.1

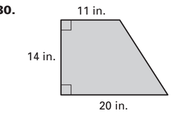
Practice B *continued*

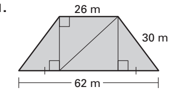
For use with the lesson "Apply the Pythagorean Theorem"

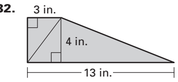
Find the area of the figure. Round decimal answers to the nearest tenth.

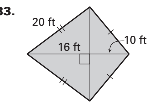
28. 

29. 

30. 

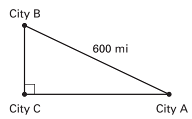
31. 

32. 

33. 

34. **Softball** In slow-pitch softball, the distance of the paths between each pair of consecutive bases is 65 feet and the paths form right angles. Find the distance the catcher must throw a softball from 3 feet behind home plate to second base.

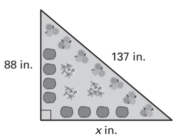
35. **Flight Distance** A small commuter airline flies to three cities whose locations form the vertices of a right triangle. The total flight distance (from city A to city B to city C and back to city A) is 1400 miles. It is 600 miles between the two cities that are furthest apart. Find the other two distances between cities.



In Exercises 36–38, use the following information.

Garden You have a garden that is in the shape of a right triangle with the dimensions shown.

36. Find the perimeter of the garden.
37. You are going to plant a post every 15 inches around the garden's perimeter. How many posts do you need?
38. You plan to attach fencing to the posts to enclose the garden. If each post costs \$1.25 and each foot of fencing costs \$.70, how much will it cost to enclose the garden? *Explain.*



LESSON 7.2

Practice B

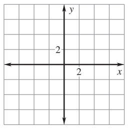
For use with the lesson "Use the Converse of the Pythagorean Theorem"

Decide whether the numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right, acute,* or *obtuse.*

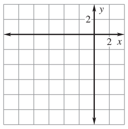
1. 5, 12, 13
2. $\sqrt{8}$, 4, 6
3. 20, 21, 28
4. 15, 36, 39
5. $\sqrt{13}$, 10, 12
6. 14, 48, 50

Graph points *A, B,* and *C*. Connect the points to form △*ABC*. Decide whether △*ABC* is *right, acute,* or *obtuse.*

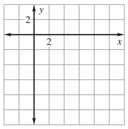
7. $A(-3, 5)$, $B(0, -2)$, $C(4, 1)$



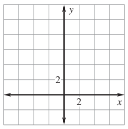
8. $A(-8, -4)$, $B(-5, -2)$, $C(-1, -7)$



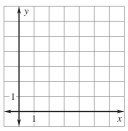
9. $A(4, 1)$, $B(7, -2)$, $C(2, -4)$



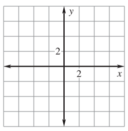
10. $A(-2, 2)$, $B(6, 4)$, $C(-4, 10)$



11. $A(0, 5)$, $B(3, 6)$, $C(5, 1)$

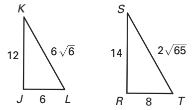


12. $A(-2, 4)$, $B(2, 0)$, $C(5, 2)$



In Exercises 13 and 14, copy and complete the statement with <, >, or =, if possible. If it is not possible, *explain* why.

13. $m\angle J$ __?__ $m\angle R$
14. $m\angle K + m\angle L$ __?__ $m\angle S + m\angle T$



LESSON 7.2

Practice B *continued*

For use with the lesson "Use the Converse of the Pythagorean Theorem"

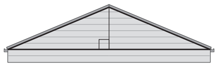
The sides and classification of a triangle are given below. The length of the longest side is the integer given. What value(s) of *x* make the triangle?

15. $x, x, 8$; right
16. $x, x, 12$; obtuse
17. $x, x, 6$; acute
18. $x, x + 3, 15$; obtuse
19. $x, x - 8, 40$; right
20. $x + 2, x + 3, 29$; acute

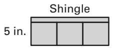
In Exercises 21 and 22, use the diagram and the following information.

Roof The roof shown in the diagram at the right is shown from the front of the house. The slope of the roof is $\frac{5}{12}$. The height of the roof is 15 feet.

21. What is the length from gutter to peak of the roof?

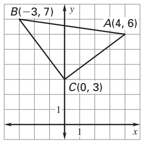


22. A row of shingles is 5 inches high. How many rows of shingles are needed for one side of the roof?



In Exercises 23–25, you will use two different methods for determining whether △*ABC* is a right triangle.

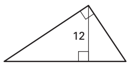
23. **Method 1** Find the slope of $\overline{AC}$ and the slope of $\overline{BC}$. What do the slopes tell you about $\angle ACB$? Is △*ABC* a right triangle? How do you know?
24. **Method 2** Use the Distance Formula and the Converse of the Pythagorean Theorem to determine whether △*ABC* is a right triangle.
25. **Compare** Which method would you use to determine whether a given triangle is right, acute, or obtuse? *Explain.*



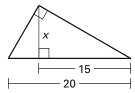
LESSON 7.3
Practice B
For use with the lesson "Use Similar Right Triangles"

Complete and solve the proportion.

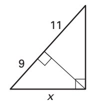
1. $\frac{x}{12} = \frac{?}{8}$



2. $\frac{15}{x} = \frac{x}{?}$



3. $\frac{9}{x} = \frac{x}{?}$



Find the value(s) of the variable(s).

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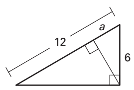

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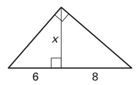

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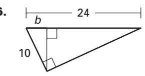

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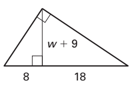

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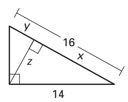

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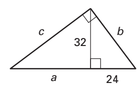

Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.

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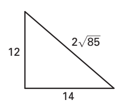

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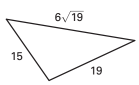

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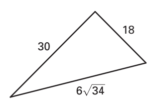

Use the Geometric Mean Theorems to find *AC* and *BD*.

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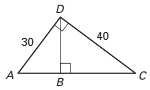

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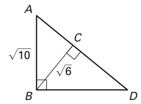

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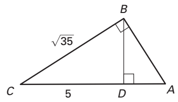


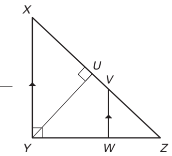
LESSON 7.3

LESSON 7.3
Practice B *continued*
For use with the lesson "Use Similar Right Triangles"

16. Complete the proof.

GIVEN: $\triangle XYZ$ is a right triangle with $m\angle XYZ = 90°$. $\overline{VW} \parallel \overline{XY}$, $\overline{YU}$ is an altitude of $\triangle XYZ$.

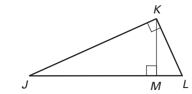
PROVE: $\triangle YUZ \sim \triangle VWZ$



Statements	Reasons
1. $\triangle XYZ$ is a right $\triangle$ with altitude $\overline{YU}$.	**1.** ___?___
2. $\triangle XYZ \sim \triangle YUZ$	**2.** ___?___
3. $\overline{VW} \parallel \overline{XY}$	**3.** ___?___
4. $\angle VWZ \cong \angle XYZ$	**4.** ___?___
5. $\angle Z \cong \angle Z$	**5.** ___?___
6. ___?___	**6.** AA Similarity Postulate
7. $\triangle YUZ \sim \triangle VWZ$	**7.** ___?___

In Exercises 17–19, use the diagram.

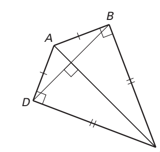
17. Sketch the three similar triangles in the diagram. Label the vertices.



18. Write similarity statements for the three triangles.

19. Which segment's length is the geometric mean of *LM* and *JM*?

20. Kite Design You are designing a diamond-shaped kite. You know that $AB = 38.4$ centimeters, $BC = 72$ centimeters, and $AC = 81.6$ centimeters. You want to use a straight crossbar $\overline{BD}$. About how long should it be?



LESSON 7.3

LESSON 7.4
Practice B
For use with the lesson "Special Right Triangles"

Find the value of *x*. Write your answer in simplest radical form.

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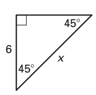
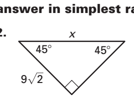


2.


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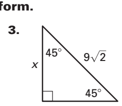

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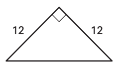

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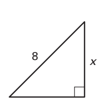

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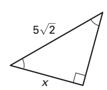
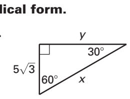


Find the value of each variable. Write your answers in simplest radical form.

7.


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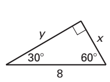

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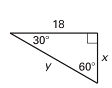

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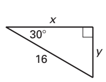

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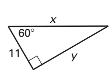

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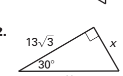
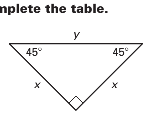
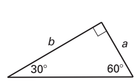


Complete the table.

13.


x	5		$\sqrt{2}$	9	
y		$4\sqrt{2}$			24

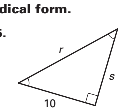
14.


a	9			11	
b		9	$5\sqrt{3}$		
c					16

LESSON 7.4

LESSON 7.4
Practice B *continued*
For use with the lesson "Special Right Triangles"

Find the value of each variable. Write your answers in simplest radical form.

15.


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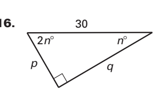

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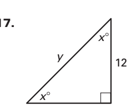

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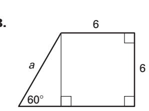

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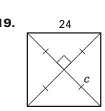
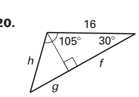


20.


The side lengths of a triangle are given. Determine whether it is a *45°-45°-90° triangle*, a *30°-60°-90° triangle*, or *neither*.

21. 5, 10, $5\sqrt{3}$

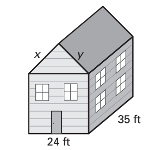
22. 7, 7, $7\sqrt{3}$

23. 6, 6, $6\sqrt{2}$

24. Roofing You are replacing the roof on the house shown, and you want to know the total area of the roof. The roof has a 1-1 pitch on both sides, which means that it slopes upward at a rate of 1 vertical unit for each 1 horizontal unit.

a. Find the values of *x* and *y* in the diagram.

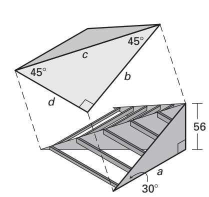
b. Find the total area of the roof to the nearest square foot.



25. Skateboard Ramp You are using wood to build a pyramid-shaped skateboard ramp. You want each ramp surface to incline at an angle of 30° and the maximum height to be 56 centimeters as shown.

a. Use the relationships shown in the diagram to determine the lengths *a*, *b*, *c*, and *d* to the nearest centimeter.

b. Suppose you want to build a second pyramid ramp with a 45° angle of incline and a maximum height of 56 inches. You can use the diagram shown by simply changing the 30° angle to 45°. Determine the lengths *a*, *b*, *c*, and *d* to the nearest centimeter for this ramp.

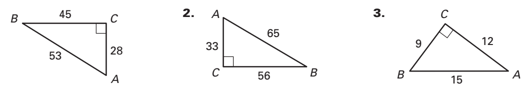


LESSON 7.4

LESSON 7.5
Practice B
For use with the lesson "Apply the Tangent Ratio"

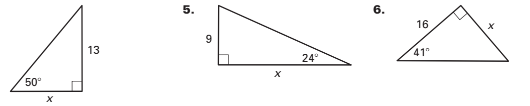
Find tan *A* and tan *B*. Write each answer as a decimal rounded to four decimal places.

1. 2. 3.

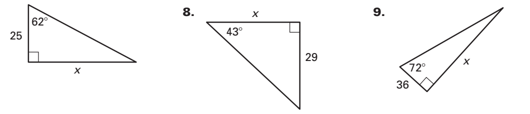


Find the value of *x* to the nearest tenth.

4. 5. 6.

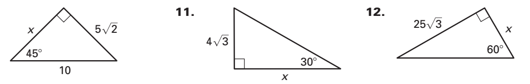


7. 8. 9.



Find the value of *x* using the definition of tangent. Then find the value of *x* using the 45°-45°-90° Triangle Theorem or the 30°-60°-90° Triangle Theorem. *Compare* the results.

10. 11. 12.

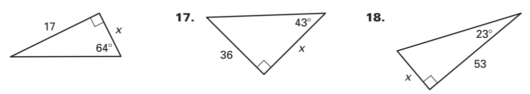


For acute ∠*A* of a right triangle, find tan *A* by using the 45°-45°-90° Triangle Theorem or the 30°-60°-90° Triangle Theorem.

13. $m\angle A = 30°$ **14.** $m\angle A = 45°$ **15.** $m\angle A = 60°$

Use a tangent ratio to find the value of *x*. Round to the nearest tenth.

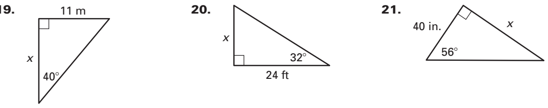
16. 17. 18.



LESSON 7.5
Practice B *continued*
For use with the lesson "Apply the Tangent Ratio"

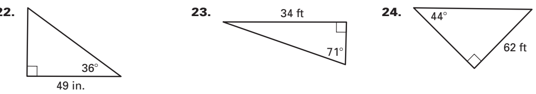
Find the area of the triangle. Round your answer to the nearest tenth.

19. 20. 21.



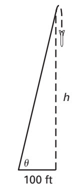
Find the perimeter of the triangle. Round to the nearest tenth.

22. 23. 24.

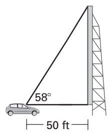


25. Model Rockets To calculate the height h reached by a model rocket, you move 100 feet from the launch point and record the angle of elevation θ to the rocket at its highest point. The values of θ for three flights are given below. Find the rocket's height to the nearest foot for the given θ in each flight.

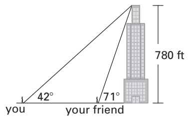
a. $\theta = 77°$
b. $\theta = 81°$
c. $\theta = 83°$



26. Drive-in Movie You are 50 feet from the screen at a drive-in movie. Your eye is on a horizontal line with the bottom of the screen and the angle of elevation to the top of the screen is 58°. How tall is the screen?



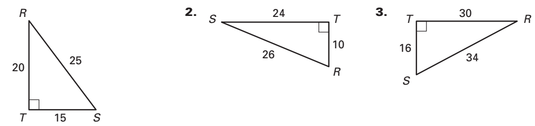
27. Skyscraper You are a block away from a skyscraper that is 780 feet tall. Your friend is between the skyscraper and yourself. The angle of elevation from your position to the top of the skyscraper is 42°. The angle of elevation from your friend's position to the top of the skyscraper is 71°. To the nearest foot, how far are you from your friend?



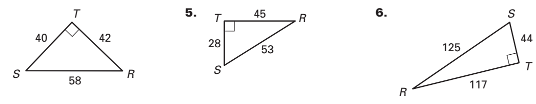
LESSON 7.6
Practice B
For use with the lesson "Apply the Sine and Cosine Ratios"

Find sin *R* and sin *S*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

1. 2. 3.

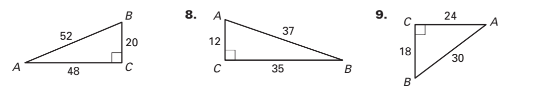


4. 5. 6.

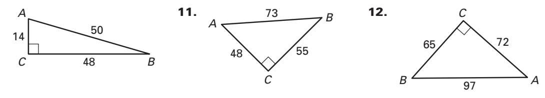


Find cos *A* and cos *B*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

7. 8. 9.

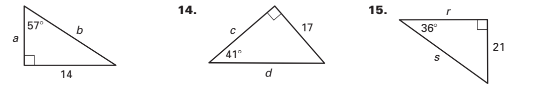


10. 11. 12.

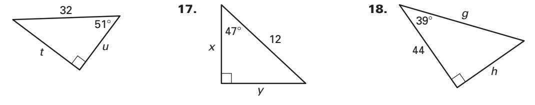


Use a cosine or sine ratio to find the value of each variable. Round decimals to the nearest tenth.

13. 14. 15.



16. 17. 18.



LESSON 7.6
Practice B *continued*
For use with the lesson "Apply the Sine and Cosine Ratios"

Use the 45°-45°-90° Triangle Theorem or the 30°-60°-90° Triangle Theorem to find the sine and cosine of the angle.

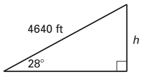
19. a 30° angle **20.** a 45° angle **21.** a 60° angle

Find the unknown side length. Then find sin *A* and cos *A*. Write each answer as a fraction in simplest form and as a decimal. Round to four decimal places, if necessary.

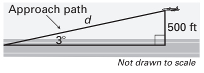
22. 23.

24. 25.

26. Ski Lift A chair lift on a ski slope has an angle of elevation of 28° and covers a total distance of 4640 feet. To the nearest foot, what is the vertical height h covered by the chair lift?

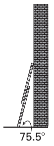


27. Airplane Landing You are preparing to land an airplane. You are on a straight line approach path that forms a 3° angle with the runway. What is the distance d along this approach path to your touchdown point when you are 500 feet above the ground? Round your answer to the nearest foot.



28. Extension Ladders You are using extension ladders to paint a chimney that is 33 feet tall. The length of an extension ladder ranges in one-foot increments from its minimum length to its maximum length. For safety, you should always use an angle of about 75.5° between the ground and the ladder.

a. Your smallest extension ladder has a maximum length of 17 feet. How high does this ladder safely reach on a vertical wall?
b. You place the base of the ladder 3 feet from the chimney. How many feet long should the ladder be?
c. To reach the top of the chimney, you need a ladder that reaches 30 feet high. How many feet long should the ladder be?



Practice B

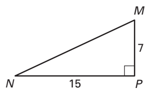
For use with the lesson "Solve Right Triangles"

Use the diagram to find the indicated measurement. Round your answer to the nearest tenth.

1. MN

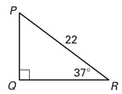
2. $m\angle M$

3. $m\angle N$



Solve the right triangle. Round decimal answers to the nearest tenth.

4.



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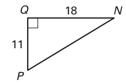

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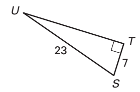

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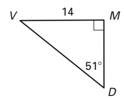

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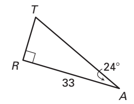

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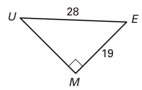

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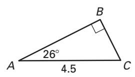
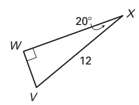
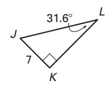


11.



12.

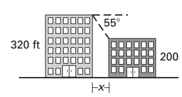


Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree.

13. $\sin A = 0.36$ **14.** $\tan A = 0.8$ **15.** $\sin A = 0.27$ **16.** $\cos A = 0.35$

17. $\tan A = 0.42$ **18.** $\cos A = 0.11$ **19.** $\sin A = 0.94$ **20.** $\cos A = 0.77$

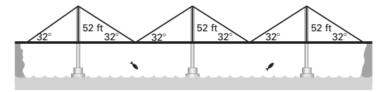
21. Office Buildings The angle of depression from the top of a 320 foot office building to the top of a 200 foot office building is 55°. How far apart are the buildings?



Practice B *continued*

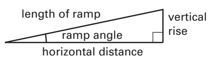
For use with the lesson "Solve Right Triangles"

22. Suspension Bridge Use the diagram to find the distance across the suspension bridge.



In Exercises 23 and 24, use the following information.

Ramps The Uniform Federal Accessibility Standards specify that the ramp angle used for a wheelchair ramp must be less than or equal to 4.78°.

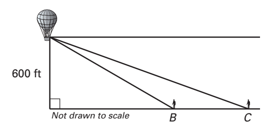


23. The length of one ramp is 16 feet. The vertical rise is 14 inches. Estimate the ramp's horizontal distance and its ramp angle. Does this ramp meet the Uniform Federal Accessibility Standards?

24. You want to build a ramp with a vertical rise of 6 inches. You want to minimize the horizontal distance taken up by the ramp. Draw a sketch showing the approximate dimensions of your ramp.

In Exercises 25–27, use the following information.

Hot Air Balloon You are in a hot air balloon that is 600 feet above the ground where you can see two people.



25. If the angle of depression from your line of sight to the person at B is 30°, how far is the person from the point on the ground below the hot air balloon?

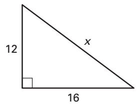
26. If the angle of depression from your line of sight to the person at C is 20°, how far is the person from the point on the ground below the hot air balloon?

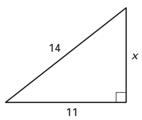
27. How far apart are the two people?

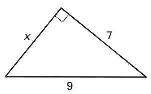
7 Assessment

CHAPTER 7 Quiz 1
For use after the lessons "Apply the Pythagorean Theorem" and "Use the Converse of the Pythagorean Theorem"

Find the unknown side length. Write your answer in simplest radical form.

1.


2.


3.


Classify the triangle formed by the side lengths as *right*, *acute*, or *obtuse*.

4. 4, 5, 6
5. 9, 12, 15
6. 11, 13, 23
7. 8, $8\sqrt{3}$, 16
8. 16, 20, 24

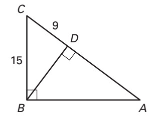
Answers

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________

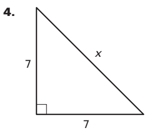
CHAPTER 7 Quiz 2
For use after the lessons "Use Similar Right Triangles" and "Special Right Triangles"

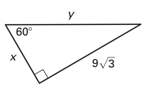
In Exercises 1–3, use the diagram.

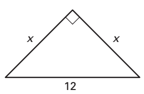
1. Find *BD*.
2. Find *AD*.
3. Find *AB*.



Find the value(s) of the variable(s). Write your answer(s) in simplest radical form.

4.


5.


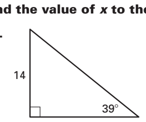
6.


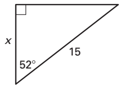
Answers

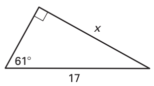
1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________

CHAPTER 7 Quiz 3
For use after the lessons "Apply the Tangent Ratio", "Apply the Sine and Cosine Ratios", and "Solve Right Triangles"

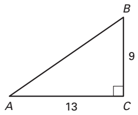
Find the value of *x* to the nearest tenth.

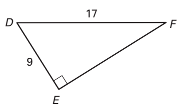
1.


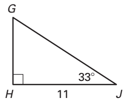
2.


3.


Solve the right triangles. Round decimal answers to the nearest tenth.

4.


5.


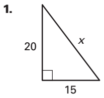
6.


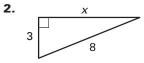
Answers

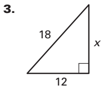
1. ____________
2. ____________
3. ____________
4. ____________
____
____
5. ____________
____
____
6. ____________
____
____

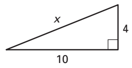
CHAPTER 7 Chapter Test B
For use after the chapter "Right Triangles and Trigonometry"

Find the value of *x*. Write your answer in simplest radical form.

1.


2.


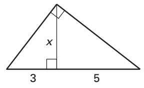
3.


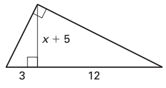
4.


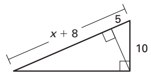
Classify the triangle as *acute*, *right*, or *obtuse*.

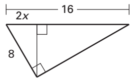
5. 5, 7, 9
6. 3, 5, $\sqrt{34}$
7. 3.1, 4.5, 5.2
8. 9, 15, $10\sqrt{3}$

Find the exact value of *x*.

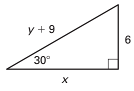
9.


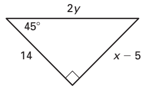
10.


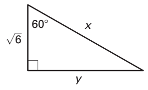
11.


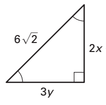
12.


Find the value of each variable. Write your answer in simplest radical form.

13.


14.


15.


16.


Answers

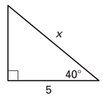
1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________
12. ____________
13. ____________
14. ____________
15. ____________
16. ____________

CHAPTER 7 **Chapter Test B** *continued*

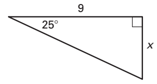
For use after the chapter "Right Triangles and Trigonometry"

Find the value of *x*. Round your answer to the nearest tenth.

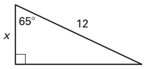
17.



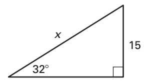
18.



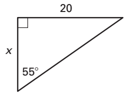
19.



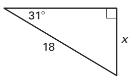
20.



21.

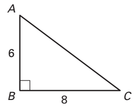


22.

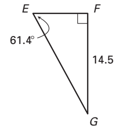


Solve the right triangle. Round your answer to the nearest tenth.

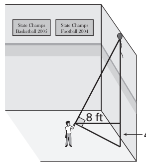
23.



24.



25. A balloon rises to the ceiling of a gymnasium. You want to find the distance from the ground to the balloon. You use a cardboard square to line up the balloon and the ground. Your friend measures the vertical distance from the ground to your eye and the distance from you to the gym wall. Approximate the distance from the ground to the balloon.



Answers

17. ____________

18. ____________

19. ____________

20. ____________

21. ____________

22. ____________

23. ____________

____

____

24. ____________

____

____

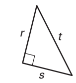
25. ____________

CHAPTER 7 **Standardized Test**

For use after the chapter "Right Triangles and Trigonometry"

Multiple Choice

1. Which equation is *not* correct?

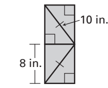


Ⓐ $t^2 - r^2 = s^2$ Ⓑ $t^2 + r^2 = s^2$
Ⓒ $s^2 - t^2 = -r^2$ Ⓓ $t^2 - s^2 = r^2$

2. A 25-foot ladder leans against a wall 7 feet from the base of the wall. How high up the wall does the ladder touch?

Ⓐ 24 ft Ⓑ 18 ft Ⓒ 20 ft Ⓓ 21.5 ft

3. Find the area of the rectangle.



Ⓐ 192 in.^{2} Ⓑ 48 in.^{2}
Ⓒ 24 in.^{2} Ⓓ 96 in.^{2}

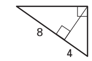
4. If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is __?__ .

Ⓐ equilateral Ⓑ a right triangle
Ⓒ acute Ⓓ none of these

5. Classify $\triangle ABC$ if the vertices are $A(-12, 5)$, $B(12, 5)$, and $C(10, 17)$.

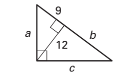
Ⓐ right scalene Ⓑ obtuse scalene
Ⓒ acute scalene Ⓓ none of these

6. Find *x*.



Ⓐ $4\sqrt{3}$ Ⓑ $2\sqrt{3}$ Ⓒ $3\sqrt{3}$ Ⓓ $5\sqrt{3}$

7. Find *a*, *b*, and *c*.

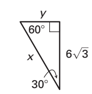


Ⓐ $a = 20, b = 25, c = 15$
Ⓑ $a = 15, b = 25, c = 20$
Ⓒ $a = 15, b = 16, c = 20$
Ⓓ $a = 16, b = 20, c = 25$

8. In a 45°-45°-90° triangle, the hypotenuse is __?__ times as long as each leg.

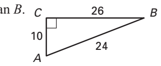
Ⓐ $\sqrt{3}$ Ⓑ $\sqrt{2}$ Ⓒ $\frac{\sqrt{2}}{2}$ Ⓓ $\frac{3}{2}$

9. Find *x* and *y*.



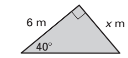
Ⓐ $x = 6, y = 12$ Ⓑ $x = 12\sqrt{3}, y = 6$
Ⓒ $x = 8\sqrt{3}, y = 8$ Ⓓ $x = 12, y = 6$

10. Find tan *A* and tan *B*.



Ⓐ $\tan A \approx 0.38, \tan B = 2.6$
Ⓑ $\tan A = 2.6, \tan B \approx 0.38$
Ⓒ $\tan A \approx 1.08, \tan B \approx 0.42$
Ⓓ $\tan A \approx 0.92, \tan B = 2.4$

11. Find the approximate area of the triangle.

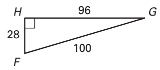


Ⓐ 15.1 m^2 Ⓑ 5.03 m^2
Ⓒ 45.3 m^2 Ⓓ 30.2 m^2

CHAPTER 7 **Standardized Test** *continued*

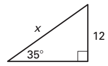
For use after the chapter "Right Triangles and Trigonometry"

12. Find sin *F* and sin *G*.



Ⓐ $\sin F = 0.28, \sin G = 0.96$
Ⓑ $\sin F \approx 3.57, \sin G \approx 1.04$
Ⓒ $\sin F \approx 1.04, \sin G \approx 3.57$
Ⓓ $\sin F = 0.96, \sin G = 0.28$

13. Which expression could be used to find the value of *x* in the diagram?

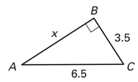


Ⓐ $\cos 55° = \frac{12}{x}$ Ⓑ $\cos 35° = \frac{x}{12}$
Ⓒ $\cos 35° = \frac{12}{x}$ Ⓓ $\cos 55° = \frac{x}{12}$

14. Which is *not* enough given information needed to solve a right triangle?

Ⓐ two acute angles and one side length
Ⓑ measure of the hypotenuse
Ⓒ two side lengths
Ⓓ one side length and the measure of one acute angle

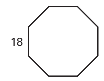
15. Find $m\angle A$.



Ⓐ 28.3° Ⓑ 32.58°
Ⓒ 57.42° Ⓓ 45°

Gridded Answer

16. Given a side length of 18 inches, find the height of a stop sign rounded to the nearest tenth.

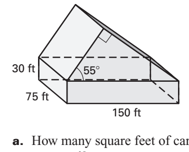


Short Response

17. You are cleaning the gutters on your house. Some bushes extend 7 feet from the wall of the house. The gutters are at a height of 24 feet. What is the minimum length of ladder you will need? If you have a 50 foot ladder, what is the minimum angle the ladder can form with the ground?

Extended Response

18. An architect creates a rough blue print for a new warehouse as shown.



a. How many square feet of carpet will you need?

b. If you paint the interior walls, how much area will you cover?

c. How many square feet of shingles will you need to cover the roof?

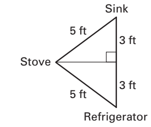
d. You decide to divide the warehouse into two rooms by building a vertical wall to the peak of the roof. What is the area of the larger room?

CHAPTER 7 **Alternative Assessment and Math Journal**

For use after the chapter "Right Triangles and Trigonometry"

Journal **1.** Describe how you would find a missing side length or a missing angle measure of a right triangle if you were given (a) the lengths of two sides or (b) the measure of one angle and the length of one side.

Multi-Step Problem **2.** A kitchen designer is planning the work triangle in a new kitchen. A work triangle is the triangle formed by the positions of the sink, the stove, and the refrigerator. The first design being considered is shown below.

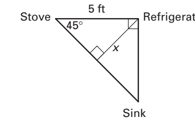


a. Classify the work triangle as *acute*, *right*, or *obtuse*.

b. Find the distance between the stove and the wall that the sink and refrigerator are on.

c. Find the measure of the angle whose vertex is the stove.

The second design being considered is shown below.



d. Find the distance from the stove to the sink. Round your answer to the nearest tenth.

e. Find the value of *x*.

f. Find the sine, cosine, and tangent of the angle whose vertex is the sink.

g. Which design do you think is better? *Explain* your reasoning.

PLAN AND PREPARE

Main Ideas

In this chapter students investigate side lengths and angles in triangles. They start by using the Pythagorean theorem to find the length of the third side in a right triangle, then use the Converse of the Pythagorean Theorem, and other theorems, to decide if three given sides lengths form an acute, right, or obtuse triangle. Students explore ratios of lengths formed by an altitude to the hypotenuse of a right triangle and use the ratios of side lengths for a 45°-45°-90° triangle and a 30°-60°-90° triangle. Finally, students apply trigonometric ratios, the Law of Sines, and the Law of Cosines to find side lengths and angle measures in triangles.

Prerequisite Skills

Skills Readiness, available online, provides review and practice for the Skills and Algebra Check portion of the Prerequisite Skills quiz.

How student answers the exercises	What to assign from *Skills Readiness*
Any of Exs. 5–8 answered incorrectly	**Skill 53** Simplify radical expressions
Any of Exs. 9–12 answered incorrectly	**Skill 77** Solve proportions
All exercises answered correctly	Chapter Enrichment

Additional skills review and practice is available in the Skills Review Handbook and the @HomeTutor.

7 Right Triangles and Trigonometry

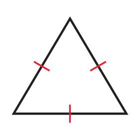
Before

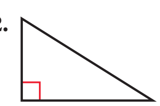
Previously, you learned the following skills, which you'll use in this chapter: classifying triangles, simplifying radicals, and solving proportions.

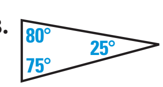
Prerequisite Skills

VOCABULARY CHECK

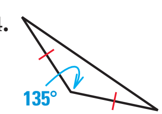
Classify the triangle shown.

1. 
equilateral

2. 
right

3. 

acute

4. 

obtuse, isosceles

SKILLS AND ALGEBRA CHECK

Simplify the radical.

5. $\sqrt{45}$ $3\sqrt{5}$ **6.** $(3\sqrt{7})^2$ 63 **7.** $\sqrt{3} \cdot \sqrt{5}$ $\sqrt{15}$ **8.** $\frac{7}{\sqrt{2}}$ $\frac{7\sqrt{2}}{2}$

Solve the proportion.

9. $\frac{3}{x} = \frac{12}{16}$ 4 **10.** $\frac{2}{3} = \frac{x}{18}$ 12 **11.** $\frac{x+5}{4} = \frac{1}{2}$ -3 **12.** $\frac{x+4}{x-4} = \frac{6}{5}$ 44

Chapter Planning Guide

Chapter Resource Book
- Teaching Guide/Lesson Plan
- Project with Rubric

Assessment and Intervention
- Assessment Book
- Benchmark Tests
- Remediation Book
- Skills Readiness

Interactive Technology
- Power Presentations
- Activity Generator
- Animated Geometry
- ExamView™ Assessment Suite
- Online Quizzes
- eEdition
- @HomeTutor

Resources for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

Now

In this chapter, you will apply the big ideas listed below and reviewed in the Chapter Summary. You will also use the key vocabulary listed below.

Big Ideas

1. **Using the Pythagorean Theorem and its converse**
2. **Using special relationships in right triangles**
3. **Using trigonometric ratios to solve right triangles**

KEY VOCABULARY

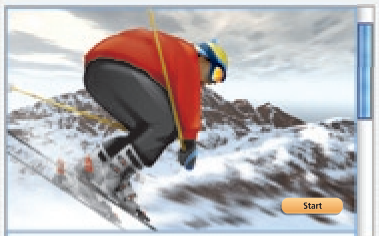
- Pythagorean triple
- trigonometric ratio
- tangent
- sine
- cosine
- angle of elevation
- angle of depression
- solve a right triangle
- inverse tangent
- inverse sine
- inverse cosine

Why?

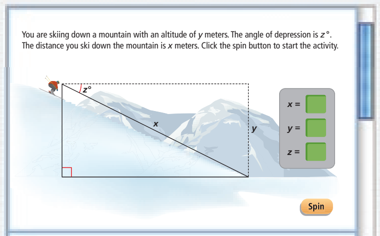
You can use trigonometric ratios to find unknown side lengths and angle measures in right triangles. For example, you can find the length of a ski slope.

Animated Geometry

The animation illustrated below helps you answer a question from this chapter: How far will you ski down the mountain?



You can use right triangles to find the distance you ski down a mountain.



Click on the "Spin" button to generate values for y and z. Find the value of x.

Animated Geometry at my.hrw.com

Differentiated Instruction Resources

- Reading Strategies
- Differentiated Instruction Lesson Notes
- English Learners Lesson Notes
- Inclusion Lesson Notes
- Teaching Strategies with Sample Worksheets
- Using Technology in the Classroom
- Tips for New Teachers
- Math Background Notes
- Assessment Strategies
- Teacher Survival Activities
- Bulletin Board Idea

1 PLAN AND PREPARE

Explore the Concept

- Students will make a tangram set to develop the Pythagorean Theorem.
- This activity leads into the study of the Pythagorean Theorem in this lesson.

Materials

Each student will need:

- graph paper
- ruler
- scissors
- Activity Support Master (*Chapter Resource Book*)

Recommended Time

Work activity: 15 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Tips for Success

Remind the students that to find the area of each square they can count the number of squares along one edge and square that number.

Alternative Strategy

Provide students with tangram sets already cut out of graph paper. Have them do Steps 3 and 4, then go over the exercises as a class.

Key Discovery

The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.

3 ASSESS AND RETEACH

1. Draw a right triangle and label the lengths of the sides a, b, and c, where c is the hypotenuse. What equation relates a, b, and c? $a^2 + b^2 = c^2$

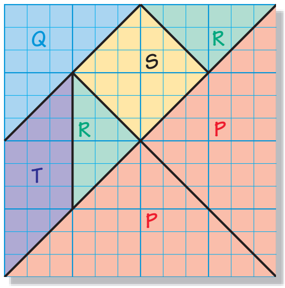
Pythagorean Theorem

MATERIALS • graph paper • ruler • pencil • scissors

QUESTION What relationship exists among the sides of a right triangle?

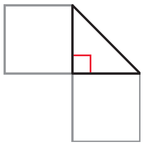
Recall that a square is a four sided figure with four right angles and four congruent sides.

EXPLORE Make and use a tangram set

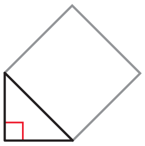


STEP 1 ***Make a tangram set*** On your graph paper, copy the tangram set as shown. Label each piece with the given letters. Cut along the solid black lines to make seven pieces.

STEP 2 ***Trace a triangle*** On another piece of paper, trace one of the large triangles P of the tangram set.



STEP 3 ***Assemble pieces along the legs*** Use all of the tangram pieces to form two squares along the legs of your triangle so that the length of each leg is equal to the side length of the square. Trace all of the pieces.



STEP 4 ***Assemble pieces along the hypotenuse*** Use all of the tangram pieces to form a square along the hypotenuse so that the side length of the square is equal to the length of the hypotenuse. Trace all of the pieces.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Find the sum of the areas of the two squares formed in Step 3. Let the letters labeling the figures represent the area of the figure. How are the side lengths of the squares related to Triangle P?
   144 units^2; the side length of the square is the same as the length of the leg of Triangle P.
2. Find the area of the square formed in Step 4. How is the side length of the square related to Triangle P? 144 units^2; the side length of the square is the same as the hypotenuse of Triangle P.
3. Compare your answers from Exercises 1 and 2. Make a conjecture about the relationship between the legs and hypotenuse of a right triangle.
4. The triangle you traced in Step 2 is an isosceles right triangle. Why? Do you think that your conjecture is true for all isosceles triangles? Do you think that your conjecture is true for all right triangles? *Justify* your answers. See margin.

3. The sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.

4. The legs of the triangles are congruent and they meet to form a right angle; no; yes; not all isosceles triangles are right triangles; for any triangle, if you construct a square on each leg and find the sum of their areas, it will be equal to the area of the square formed on the hypotenuse.

7.1 Apply the Pythagorean Theorem

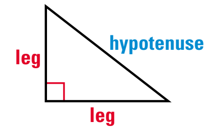
Before You learned about the relationships within triangles.

Now You will find side lengths in right triangles.

Why? So you can find the shortest distance to a campfire, as in Ex. 35.

Key Vocabulary
- Pythagorean triple
- right triangle
- leg of a right triangle
- hypotenuse

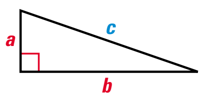
One of the most famous theorems in mathematics is the Pythagorean Theorem, named for the ancient Greek mathematician Pythagoras (around 500 B.C.). This theorem can be used to find information about the lengths of the sides of a right triangle.



THEOREM *For Your Notebook*

THEOREM 7.1 Pythagorean Theorem

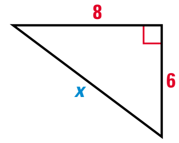
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

EXAMPLE 1 Find the length of a hypotenuse

Find the length of the hypotenuse of the right triangle.



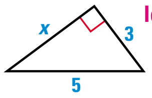
Solution

ABBREVIATE
In the equation for the Pythagorean Theorem, "length of hypotenuse" and "length of leg" was shortened to "hypotenuse" and "leg".

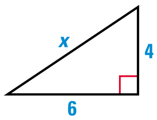
$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Pythagorean Theorem
$x^2 = 6^2 + 8^2$	Substitute.
$x^2 = 36 + 64$	Multiply.
$x^2 = 100$	Add.
$x = 10$	Find the positive square root.

GUIDED PRACTICE for Example 1

Identify the unknown side as a *leg* or *hypotenuse*. Then, find the unknown side length of the right triangle. Write your answer in simplest radical form.

1. 

leg; 4

2. 

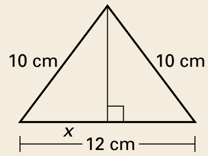
hypotenuse; $2\sqrt{13}$

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. Solve $x^2 = 100$. $10, -10$
2. Solve $x^2 + 9 = 25$ $4, -4$
3. Simplify $\sqrt{20}$. $2\sqrt{5}$
4. Find x. 6 cm



Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
- See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

If you know the lengths of two sides of a right triangle, how do you find the length of the third side? Tell students they will learn how to answer this question by using the Pythagorean Theorem.

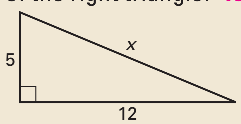
Motivating the Lesson

You are at one corner of a football field and your friend is at the opposite corner. Is it shorter to walk diagonally across the field or to walk around it? In this lesson students will learn how to find the length of a diagonal of a rectangle.

3 TEACH

Extra Example 1

Find the length of the hypotenuse of the right triangle. **13**



Extra Example 2

Randy made a ramp for his dog to get into his truck. The ramp is 6 feet long and the bed of the truck is 3 feet above the ground. Approximately how far from the back of the truck does the ramp touch the ground? **C**

Ⓐ 3 ft　　Ⓑ 4 ft

Ⓒ 5.2 ft　　Ⓓ 6.7 ft

Animated Geometry my.hrw.com

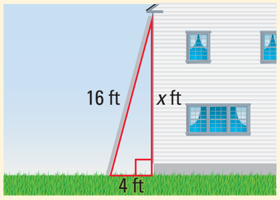
An **Animated Geometry** activity is available online for **Proving the Pythagorean Theorem**. This activity is also part of **Power Presentations**.

★ EXAMPLE 2 Standardized Test Practice

A 16 foot ladder rests against the side of the house, and the base of the ladder is 4 feet away. Approximately how high above the ground is the top of the ladder?

Ⓐ 240 feet　　Ⓑ 20 feet

Ⓒ 16.5 feet　　Ⓓ 15.5 feet



Solution

$$\left(\text{Length of ladder}\right)^2 = \left(\text{Distance from house}\right)^2 + \left(\text{Height of ladder}\right)^2$$

$16^2 = 4^2 + x^2$	**Substitute.**
$256 = 16 + x^2$	**Multiply.**
$240 = x^2$	**Subtract 16 from each side.**
$\sqrt{240} = x$	**Find positive square root.**
$15.492 \approx x$	**Approximate with a calculator.**

APPROXIMATE
In real-world applications, it is usually appropriate to use a calculator to approximate the square root of a number. Round your answer to the nearest tenth.

The ladder is resting against the house at about 15.5 feet above the ground.

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

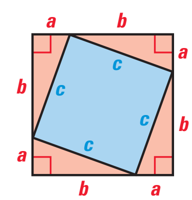
✓ GUIDED PRACTICE for Example 2

3. The top of a ladder rests against a wall, 23 feet above the ground. The base of the ladder is 6 feet away from the wall. What is the length of the ladder? **about 23.8 ft**

4. The Pythagorean Theorem is only true for what type of triangle? **right triangle**

PROVING THE PYTHAGOREAN THEOREM There are many proofs of the Pythagorean Theorem. An informal proof is shown below.

In the figure at the right, the four right triangles are congruent, and they form a small square in the middle. The area of the large square is equal to the area of the four triangles plus the area of the smaller square.



$$\text{Area of large square} = \text{Area of four triangles} + \text{Area of smaller square}$$

$(a + b)^2 = 4\left(\frac{1}{2}ab\right) + c^2$	**Use area formulas.**
$a^2 + 2ab + b^2 = 2ab + c^2$	**Multiply.**
$a^2 + b^2 = c^2$	**Subtract $2ab$ from each side.**

REVIEW AREA
Recall that the area of a square with side length s is $A = s^2$. The area of a triangle with base b and height h is $A = \frac{1}{2}bh$.

Animated Geometry at my.hrw.com

Differentiated Instruction

Kinesthetic Learners After discussing **Example 2**, set up a similar problem in the classroom. For example, have students find the height of a stack of textbooks indirectly by leaning a ruler from the desktop to the top of the stack. Have them measure the distance from the base of the stack of books to the bottom of the ruler. Instruct them to use the Pythagorean Theorem to find the height of the stack of books.

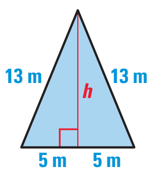
See also the *Differentiated Instruction Resources* for more strategies.

EXAMPLE 3 Find the area of an isosceles triangle

Find the area of the isosceles triangle with side lengths 10 meters, 13 meters, and 13 meters.

Solution

STEP 1 **Draw** a sketch. By definition, the length of an altitude is the height of a triangle. In an isosceles triangle, the altitude to the base is also a perpendicular bisector. So, the altitude divides the triangle into two right triangles with the dimensions shown.



STEP 2 **Use** the Pythagorean Theorem to find the height of the triangle.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$13^2 = 5^2 + h^2$	Substitute.
$169 = 25 + h^2$	Multiply.
$144 = h^2$	Subtract 25 from each side.
$12 = h$	Find the positive square root.

STEP 3 **Find** the area.

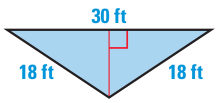
$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(10)(12) = 60 \text{ m}^2$$

▶ The area of the triangle is 60 square meters.

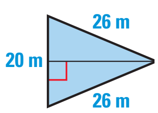
READ TABLES
You may find it helpful to use the Table of Squares and Square Roots on T6.

✓ GUIDED PRACTICE for Example 3

Find the area of the triangle.

5. 

about 149.2 ft^2

6. 

240 m^2

PYTHAGOREAN TRIPLES A ____________ is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

KEY CONCEPT *For Your Notebook*

Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

STANDARDIZED TESTS
You may find it helpful to memorize the basic Pythagorean triples, shown in **bold**, for standardized tests.

Extra Example 3

Find the area of the isosceles triangle with side lengths 20 in., 20 in., and 24 in. **192 in.^{2}**

Key Question to Ask for Example 3

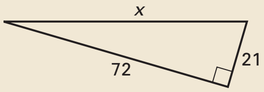
- How do you know that the altitude bisects the base? ***Sample answer:*** **The vertex from which the altitude to the base of an isosceles triangle is drawn is equidistant from the endpoints of the base. By the Converse of the Perpendicular Bisector Theorem, the vertex lies on the perpendicular bisector of the base.**

Avoiding Common Errors

In the Key Concept about Pythagorean triples, be sure students understand that if the two legs of a right triangle have lengths 3 and 5, the hypotenuse is *not* 4. The hypotenuse must be the longest side of the triangle.

Extra Example 4

Find the length of the hypotenuse of the right triangle. **75**



Key Question to Ask for Example 4

- If the legs were 2.5 and 6, how could you find the length of the hypotenuse without using the Pythagorean Theorem? **Using the Pythagorean triple 5, 12, 13, the lengths 2.5 and 6 are half of 5 and 12, so the hypotenuse must be half of 13.**

Closing the Lesson

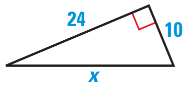
Have students summarize the major points of the lesson and answer the Essential Question: If you know the lengths of two sides of a right triangle, how do you find the length of the third side?

- **In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs.**
- **A Pythagorean triple, such as 3, 4, 5 or 5, 12, 13, is a set of three positive integers that satisfy the Pythagorean Theorem.**

If you know the lengths of two sides of a right triangle, use the Pythagorean Theorem or a Pythagorean triple to find the length of the third side.

EXAMPLE 4 Find the length of a hypotenuse using two methods

Find the length of the hypotenuse of the right triangle.



Solution

Method 1: Use a Pythagorean triple.

A common Pythagorean triple is 5, 12, 13. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 2, you get the lengths of the legs of this triangle: $5 \cdot 2 = 10$ and $12 \cdot 2 = 24$. So, the length of the hypotenuse is $13 \cdot 2 = 26$.

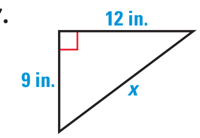
Method 2: Use the Pythagorean Theorem.

$x^2 = 10^2 + 24^2$	Pythagorean Theorem
$x^2 = 100 + 576$	Multiply.
$x^2 = 676$	Add.
$x = 26$	Find the positive square root.

✓ GUIDED PRACTICE for Example 4

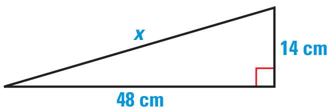
Find the unknown side length of the right triangle using the Pythagorean Theorem. Then use a Pythagorean triple.

7.



15 in.

8.



50 cm

7.1 EXERCISES

HOMEWORK KEY

○ = See WORKED-OUT SOLUTIONS Exs. 9, 11, and 33

★ = STANDARDIZED TEST PRACTICE Exs. 2, 17, 27, 33, and 36

◆ = MULTIPLE REPRESENTATIONS Ex. 35

SKILL PRACTICE

A

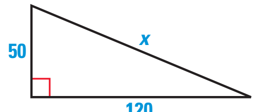
1. **VOCABULARY** Copy and complete: A set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$ is called a _?_. **Pythagorean triple**

2. ★ **WRITING** *Describe* the information you need to have in order to use the Pythagorean Theorem to find the length of a side of a triangle. **A right triangle, the measure of a leg of the triangle, and the measure of either the hypotenuse or the other leg.**

EXAMPLE 1 for Exs. 3–7

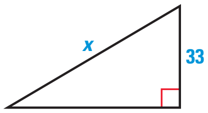
ALGEBRA **Find the length of the hypotenuse of the right triangle.**

3.



130

4.



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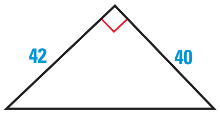
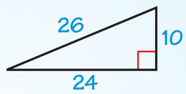
5.



58

ERROR ANALYSIS *Describe* and correct the error in using the Pythagorean Theorem.

6.

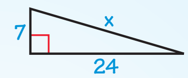


$a^2 + b^2 = c^2$

$10^2 + 26^2 = 24^2$

a and *b* represent the legs of the triangle, but 26 is the hypotenuse; $10^2 + 24^2 = 26^2$.

7.



$x^2 = 7^2 + 24^2$

$x^2 = (7 + 24)^2$

$x^2 = 31^2$

$x = 31$

In step 2, the Distributive Property was used incorrectly; $x^2 = 49 + 576$, $x^2 = 625$, $x = 25$.

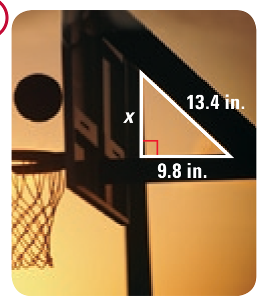
EXAMPLE 2 for Exs. 8–10

FINDING A LENGTH Find the unknown leg length *x*.

8.

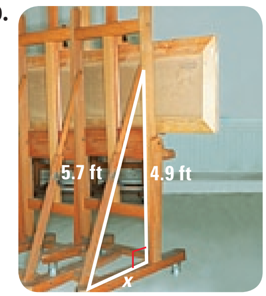
about 14.1 ft

9.



about 9.14 in.

10.

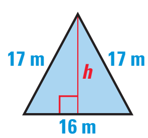


about 2.91 ft

EXAMPLE 3 for Exs. 11–13

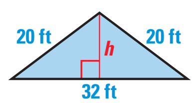
FINDING THE AREA Find the area of the isosceles triangle.

11.



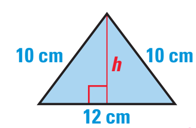
120 m^2

12.



192 ft^2

13.

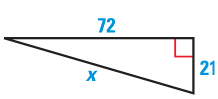


48 cm^2

EXAMPLE 4 for Exs. 14–17

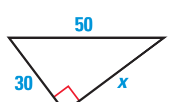
FINDING SIDE LENGTHS Find the unknown side length of the right triangle using the Pythagorean Theorem or a Pythagorean triple.

14.



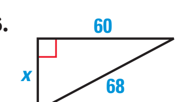
75

15.



40

16.



32

17. ★ **MULTIPLE CHOICE** What is the length of the hypotenuse of a right triangle with leg lengths of 8 inches and 15 inches? **B**

Ⓐ 13 inches Ⓑ 17 inches Ⓒ 21 inches Ⓓ 25 inches

PYTHAGOREAN TRIPLES The given lengths are two sides of a right triangle. All three side lengths of the triangle are integers and together form a Pythagorean triple. Find the length of the third side and tell whether it is a leg or the hypotenuse.

B

18. 24 and 51 **45, leg** **19.** 20 and 25 **15, leg** **20.** 28 and 96 **100, hypotenuse**

21. 20 and 48 **52, hypotenuse** **22.** 75 and 85 **40, leg** **23.** 72 and 75 **21, leg**

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1: SRH p. SR6 Exs. 1–4, 6, 7
Exs. 1–10, 31–33
Day 2:
Exs. 11–23, 34, 35

Average:
Day 1:
Exs. 1, 2, 4–9, 24–26, 31–33
Day 2:
Exs. 12, 13, 15–17, 20–22, 27, 28, 34–37

Advanced:
Day 1:
Exs. 1, 2, 4, 5, 9, 10, 24–26, 31–33, 36–38*
Day 2:
Exs. 13, 16, 17, 21–23, 27–30*, 34, 35

Block:
Exs. 1, 2, 4–9, 12, 13, 15–17, 20–22, 24–28, 31–37

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 8, 12, 14, 31
Average: 4, 8, 12, 15, 32
Advanced: 5, 10, 13, 16, 34

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

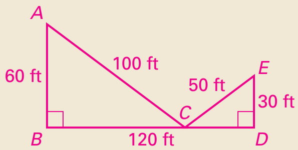
An easily-readable reduced practice page can be found at the beginning of this chapter.

Study Strategy

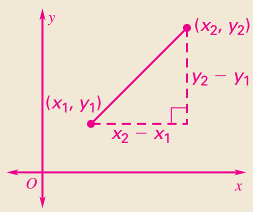
Exercises 3–5, 11–17 Identify which exercises use Pythagorean triples or their multiples. For example, the lengths in Exercise 3 are 10 times the numbers in the Pythagorean triple 5, 12, 13.

35a–b. See below.

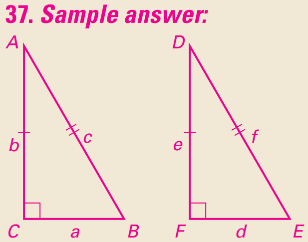
35c.



36.



37. ***Sample answer:***



Given: $\triangle ABC$ and $\triangle DEF$ are right triangles; $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$.

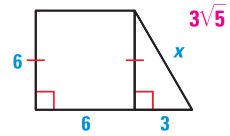
Prove: $\triangle ABC \cong \triangle DEF$

Statements (Reasons)

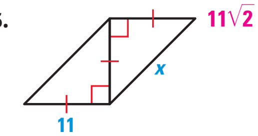
1. $\triangle ABC$ and $\triangle DEF$ are right triangles; $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$. (Given)
2. $c = f$, $b = e$ (Definition of segment congruence)
3. $a^2 + b^2 = c^2$, $d^2 + e^2 = f^2$ (Pythagorean Theorem)
4. $a^2 + b^2 = f^2$ (Substitution Property of Equality)
5. $a^2 + b^2 = d^2 + e^2$ (Substitution Property of Equality)
6. $a^2 + e^2 = d^2 + e^2$ (Substitution Property of Equality)
7. $a^2 = d^2$ (Subtraction Property of Equality)
8. $a = d$ (A property of square roots)
9. $\overline{BC} \cong \overline{EF}$ (Definition of segment congruence)
10. $\angle C \cong \angle F$ (Right Angle Congruence Theorem)
11. $\triangle ABC \cong \triangle DEF$ (SAS Congruence Postulate)

FINDING SIDE LENGTHS **Find the unknown side length x. Write your answer in simplest radical form.**

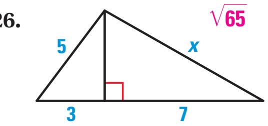
24. $3\sqrt{5}$



25. $11\sqrt{2}$



26. $\sqrt{65}$



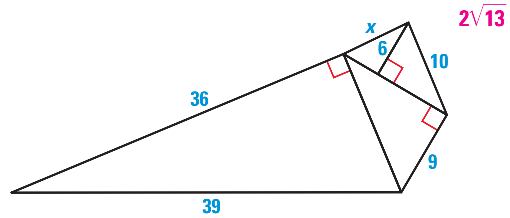
27. ★ **MULTIPLE CHOICE** What is the area of a right triangle with a leg length of 15 feet and a hypotenuse length of 39 feet? **A**

Ⓐ 270 ft^2 Ⓑ 292.5 ft^2 Ⓒ 540 ft^2 Ⓓ 585 ft^2

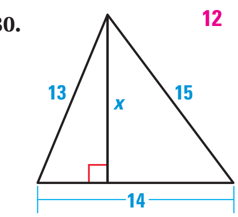
28. **ALGEBRA** Solve for x if the lengths of the two legs of a right triangle are $2x$ and $2x + 4$, and the length of the hypotenuse is $4x - 4$. **6**

CHALLENGE **In Exercises 29 and 30, solve for x.**

C **29.** $2\sqrt{13}$



30. 12

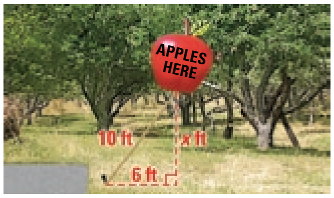


PROBLEM SOLVING

EXAMPLE 2 A for Exs. 31–32

31. **BASEBALL DIAMOND** In baseball, the distance of the paths between each pair of consecutive bases is 90 feet and the paths form right angles. How far does the ball need to travel if it is thrown from home plate directly to second base? **about 127 ft**

32. **APPLE BALLOON** You tie an apple balloon to a stake in the ground. The rope is 10 feet long. As the wind picks up, you observe that the balloon is now 6 feet away from the stake. How far above the ground is the balloon now? **8 ft**



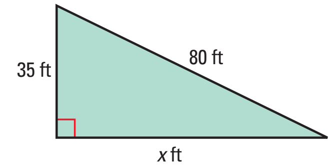
33. ★ **SHORT RESPONSE** Three side lengths of a right triangle are 25, 65, and 60. *Explain* how you know which side is the hypotenuse. **The longest side of the triangle is opposite the largest angle, which in a right triangle is the right angle.**

34. **MULTI-STEP PROBLEM** In your town, there is a field that is in the shape of a right triangle with the dimensions shown.

a. Find the perimeter of the field. **about 187 ft**

b. You are going to plant dogwood seedlings about every ten feet around the field's edge. How many trees do you need? **about 19 trees**

c. If each dogwood seedling sells for $12, how much will the trees cost? **about $228**

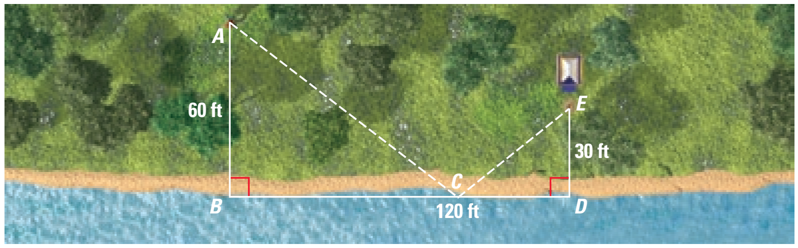


◯ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE** ◆ = **MULTIPLE REPRESENTATIONS**

35a–b.

BC	10	20	30	40	50	60	70	80	90	100	110	120
AC	60.8	63.2	67.1	72.1	78.1	84.9	92.2	100	108.2	116.6	125.3	134.2
CE	114.0	104.4	94.9	85.4	76.2	67.1	58.3	50	42.4	36.1	31.6	30
AC + *CE*	174.8	167.6	162	157.5	154.3	152	150.5	150	150.6	152.7	156.9	164.2

B **35.** **MULTIPLE REPRESENTATIONS** As you are gathering leaves for a science project, you look back at your campsite and see that the campfire is not completely out. You want to get water from a nearby river to put out the flames with the bucket you are using to collect leaves. Use the diagram and the steps below to determine the shortest distance you must travel.



a. Making a Table Make a table with columns labeled BC, AC, CE, and $AC + CE$. Enter values of BC from 10 to 120 in increments of 10. **See margin.**

b. Calculating Values Calculate AC, CE, and $AC + CE$ for each value of BC, and record the results in the table. Then, use your table of values to determine the shortest distance you must travel. **150 ft**

c. Drawing a Picture Draw an accurate picture to scale of the shortest distance. **See margin.**

36. ★ **SHORT RESPONSE** *Justify* the Distance Formula using the Pythagorean Theorem. **See margin for art; by the Pythagorean Theorem, $(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$ so $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.**

37. **PROVING A THEOREM** Find the Hypotenuse-Leg (HL) Congruence Theorem. Assign variables for the side lengths in the diagram. Use your variables to write GIVEN and PROVE statements. Use the Pythagorean Theorem and congruent triangles to prove the Hypotenuse-Leg (HL) Congruence Theorem. **See margin.**

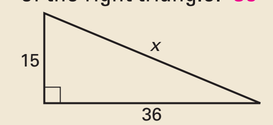
C **38.** **CHALLENGE** Trees grown for sale at nurseries should stand at least five feet from one another while growing. If the trees are grown in parallel rows, what is the smallest allowable distance between rows? **about 4.3 ft**

5 ASSESS AND RETEACH

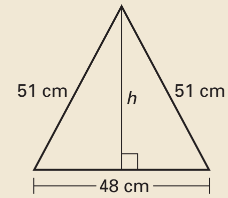
Daily Homework Quiz

Also available online

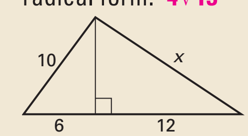
1. Find the length of the hypotenuse of the right triangle. **39**



2. Find the area of the isosceles triangle. **1080 cm^2**



3. Find the unknown side length x. Write your answer in simplest radical form. **$4\sqrt{13}$**



Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

my.hrw.com Keystrokes

1 PLAN AND PREPARE

Explore the Concept

- Students will use the Converse of the Pythagorean Theorem to classify a triangle.
- This leads into using the Converse of the Pythagorean Theorem in this lesson.

Materials

Each student will need:

- graphing calculator or computer with geometry software

Recommended Time

Work activity: 15 min

Discuss results: 5 min

Grouping

Students can work individually or in pairs. If students work in pairs, one can measure while the other records the results.

2 TEACH

Tips for Success

If possible, let the software do the calculations and print results.

Alternative Strategy

Do the activity as a demonstration on a computer. Have students take turns doing the calculations.

Key Discovery

Compare AB^2 to $AC^2 + CB^2$. If AB^2 is greater, the triangle is obtuse; if AB^2 is less, the triangle is acute; if AB^2 is equal, the triangle is right.

3 ASSESS AND RETEACH

1. If $AB^2 = 31$ and $AC^2 + CB^2 = 26$, what type of triangle is $\triangle ABC$? **obtuse**

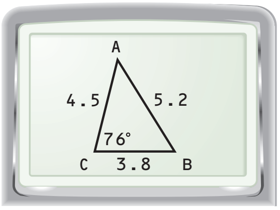
Converse of the Pythagorean Theorem

MATERIALS • graphing calculator or computer

QUESTION **How can you use the side lengths in a triangle to classify the triangle by its angle measures?**

You can use geometry drawing software to construct and measure triangles.

EXPLORE **Construct a triangle**



STEP 1 ***Draw a triangle*** Draw any $\triangle ABC$ with the largest angle at C. Measure $\angle C$, $\overline{AB}$, $\overline{AC}$, and $\overline{CB}$. **Check students' work.**

STEP 2 ***Calculate*** Use your measurements to calculate AB^2, AC^2, CB^2, and $(AC^2 + CB^2)$. **Check students' work.**

STEP 3 ***Complete a table*** Copy the table below and record your results in the first row. Then move point A to different locations and record the values for each triangle in your table. Make sure $\overline{AB}$ is always the longest side of the triangle. Include triangles that are acute, right, and obtuse. **Check students' work.**

$m\angle C$	AB	AB^2	AC	CB	$AC^2 + CB^2$
76°	5.2	27.04	4.5	3.8	34.69
?	?	?	?	?	?
?	?	?	?	?	?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. The Pythagorean Theorem states that "In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs." Write the Pythagorean Theorem in if-then form. Then write its converse. **See margin.**
2. Is the converse of the Pythagorean Theorem true? *Explain.* **Yes; because the theorem can be stated as an equation, it will be true in either direction.**
3. Make a conjecture about the relationship between the measure of the largest angle in a triangle and the squares of the side lengths. **Check students' work.**

Copy and complete the statement.

4. If $AB^2 > AC^2 + CB^2$, then the triangle is a(n) __?__ triangle. **obtuse**
5. If $AB^2 < AC^2 + CB^2$, then the triangle is a(n) __?__ triangle. **acute**
6. If $AB^2 = AC^2 + CB^2$, then the triangle is a(n) __?__ triangle. **right**

1. If a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs; if the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs, then the triangle is a right triangle.

7.2 Use the Converse of the Pythagorean Theorem

Before You used the Pythagorean Theorem to find missing side lengths.

Now You will use its converse to determine if a triangle is a right triangle.

Why? So you can determine if a volleyball net is set up correctly, as in Ex. 38.

Key Vocabulary
- acute triangle
- obtuse triangle

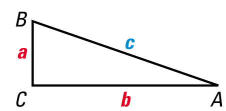
The converse of the Pythagorean Theorem is also true. You can use it to verify that a triangle with given side lengths is a right triangle.

THEOREM *For Your Notebook*

THEOREM 7.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

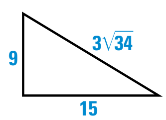
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.



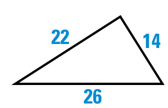
EXAMPLE 1 Verify right triangles

Tell whether the given triangle is a right triangle.

a.



b.



Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(3\sqrt{34})^2 \stackrel{?}{=} 9^2 + 15^2$

$9 \cdot 34 \stackrel{?}{=} 81 + 225$

$306 = 306$ ✓

The triangle is a right triangle.

b. $26^2 \stackrel{?}{=} 22^2 + 14^2$

$676 \stackrel{?}{=} 484 + 196$

$676 \neq 680$

The triangle is not a right triangle.

REVIEW ALGEBRA
Use a square root table or a calculator to find the decimal representation. So, $3\sqrt{34} \approx 17.493$ is the length of the longest side in part (a).

✓ **GUIDED PRACTICE** for Example 1

Tell whether a triangle with the given side lengths is a right triangle.

1. 4, $4\sqrt{3}$, 8 **right triangle**

2. 10, 11, and 14 **not a right triangle**

3. 5, 6, and $\sqrt{61}$ **right triangle**

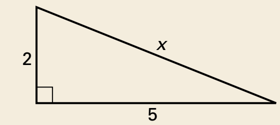
HMH Photo

1 PLAN AND PREPARE

Warm-Up Exercises

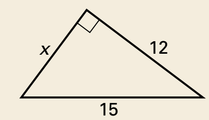
Also available online

1. Find x. $\sqrt{29}$



2. Simplify $(5\sqrt{3})^2$. **75**

3. Find x. **9**



Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day

Average: 1 day

Advanced: 1 day

Block: 0.5 block with next lesson

- See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How can you use the sides of a triangle to determine if it is right?

Tell students they will learn how to answer this question by using theorems about the sides of a triangle.

Motivating the Lesson

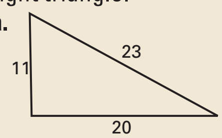
You want to put in a fence post and be sure it is perpendicular to the ground. In this lesson you will learn how to use the lengths of sides of a triangle to determine whether or not one side is perpendicular to another side.

3 TEACH

Extra Example 1

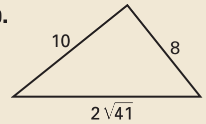
Tell whether the given triangle is a right triangle.

a.



No; 521 ≠ 529.

b.



Yes; 164 = 164.

Extra Example 2

Can segments with lengths of 11.2 inches, 6.5 inches, and 7.1 inches form a triangle? If so, would the triangle be acute, right, or obtuse? **Yes; obtuse**

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

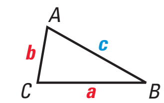
CLASSIFYING TRIANGLES The Converse of the Pythagorean Theorem is used to verify that a given triangle is a right triangle. The theorems below are used to verify that a given triangle is acute or obtuse.

THEOREMS *For Your Notebook*

THEOREM 7.3

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an acute triangle.

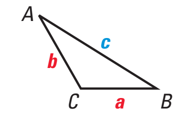
If $c^2 < a^2 + b^2$, then the triangle is acute.



THEOREM 7.4

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is an obtuse triangle.

If $c^2 > a^2 + b^2$, then triangle ABC is obtuse.



EXAMPLE 2 Classify triangles

Can segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle? If so, would the triangle be *acute, right,* or *obtuse*?

APPLY THEOREMS
The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Solution

STEP 1 **Use** the Triangle Inequality Theorem to check that the segments can make a triangle.

$4.3 + 5.2 = 9.5$	$4.3 + 6.1 = 10.4$	$5.2 + 6.1 = 11.3$
$9.5 > 6.1$	$10.4 > 5.2$	$11.3 > 4.3$

▶ The side lengths 4.3 feet, 5.2 feet, and 6.1 feet can form a triangle.

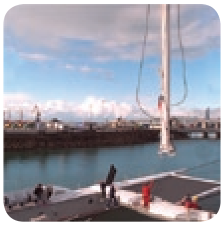
STEP 2 **Classify** the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.

$c^2 \;\underline{\;?\;}\; a^2 + b^2$	**Compare c^2 with $a^2 + b^2$.**
$6.1^2 \;\underline{\;?\;}\; 4.3^2 + 5.2^2$	**Substitute.**
$37.21 \;\underline{\;?\;}\; 18.49 + 27.04$	**Simplify.**
$37.21 < 45.53$	**c^2 is less than $a^2 + b^2$.**

▶ The side lengths 4.3 feet, 5.2 feet, and 6.1 feet form an acute triangle.

Animated Geometry at my.hrw.com

EXAMPLE 3 Use the Converse of the Pythagorean Theorem

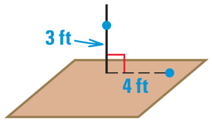


CATAMARAN You are part of a crew that is installing the mast on a catamaran. When the mast is fastened properly, it is perpendicular to the trampoline deck. How can you check that the mast is perpendicular using a tape measure?

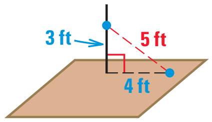
Solution

To show a line is perpendicular to a plane you must show that the line is perpendicular to two lines in the plane.

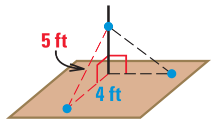
Think of the mast as a line and the deck as a plane. Use a 3-4-5 right triangle and the Converse of the Pythagorean Theorem to show that the mast is perpendicular to different lines on the deck.



First place a mark 3 feet up the mast and a mark on the deck 4 feet from the mast.



Use the tape measure to check that the distance between the two marks is 5 feet. The mast makes a right angle with the line on the deck.



Finally, repeat the procedure to show that the mast is perpendicular to another line on the deck.

✓ GUIDED PRACTICE for Examples 2 and 3

4. Show that segments with lengths 3, 4, and 6 can form a triangle and classify the triangle as *acute*, *right*, or *obtuse*. **$3 + 4 > 6$, $4 + 6 > 3$, $6 + 3 > 4$, obtuse**

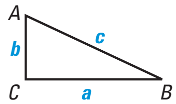
5. WHAT IF? In Example 3, could you use triangles with side lengths 2, 3, and 4 to verify that you have perpendicular lines? *Explain.* **No; in order to verify that you have perpendicular lines, the triangle would have to be a right triangle, and a 2-3-4 triangle is not a right triangle.**

CLASSIFYING TRIANGLES You can use the theorems from this lesson to classify a triangle as acute, right, or obtuse based on its side lengths.

CONCEPT SUMMARY *For Your Notebook*

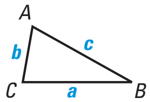
Methods for Classifying a Triangle by Angles Using its Side Lengths

Theorem 7.2



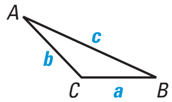
If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.

Theorem 7.3



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.

Theorem 7.4



If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

Extra Example 3

You are planting a tree and want it to be perpendicular to the ground. How can you check this using a meter stick? ***Sample answer:*** **Mark a point 80 cm off the ground on the tree and put several stakes in the ground 60 cm from the base of the tree. Adjust the angle of the tree until the 80-cm mark is exactly 100 cm from each of the stakes.**

Key Questions to Ask for Example 3

- How do you show that a line is perpendicular to a plane? **Show that it is perpendicular to 2 lines in the plane.**
- What are the lengths of the segments on the mast and on the deck? **3 ft and 4 ft**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How can you use the sides of a triangle to determine if it is right?

- **Let c be the length of the longest side and let a and b be the lengths of the other two sides. Then:**
  If $c^2 < a^2 + b^2$, the triangle is an acute triangle.
  If $c^2 = a^2 + b^2$, the triangle is a right triangle.
  If $c^2 > a^2 + b^2$, the triangle is an obtuse triangle.

A triangle is a right triangle if the square of its longest side is equal to the sum of the squares of its two other sides.

Differentiated Instruction

Kinesthetic Learners Create a model for **Example 3** using a piece of cardboard, a pencil, and two 5-inch long pieces of string. The cardboard will be the boat and the pencil will be the mast. Attach the strings 3 inches up from one end of the pencil. Then attach each end to two separate points on the cardboard, 4 inches away from where the pencil rests on the cardboard. Students should see that the pencil must be perpendicular to the cardboard.

See also the *Differentiated Instruction Resources* for more strategies.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–7, 9–11, 15–18, 24–28, 35–40

Average:
Day 1:
Exs. 1, 2, 5–7, 10–12, 18–20, 24–31, 35–44

Advanced:
Day 1:
Exs. 1, 2, 7, 8, 12–14, 21–25, 27–35*, 37–45*

Block:
Exs. 1, 2, 5–7, 10–12, 18–20, 24–31, 35–44 (with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 10, 16, 35, 36
Average: 6, 12, 20, 35, 38
Advanced: 8, 14, 22, 35, 40

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

7.2 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 7, 17, and 37
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 24, 25, 32, 38, 39, and 43

SKILL PRACTICE

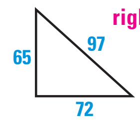
A

1. VOCABULARY What is the longest side of a right triangle called? **hypotenuse**

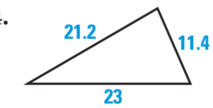
2. ★ WRITING *Explain* how the side lengths of a triangle can be used to classify it as acute, right, or obtuse. **See margin.**

EXAMPLE 1 for Exs. 3–14

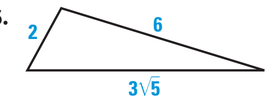
VERIFYING RIGHT TRIANGLES **Tell whether the triangle is a right triangle.**

3. 

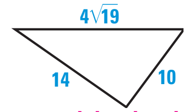
right triangle

4. 

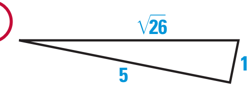
not a right triangle

5. 

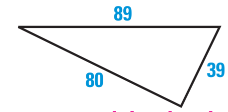
not a right triangle

6. 

not a right triangle

7. 

right triangle

8. 

right triangle

VERIFYING RIGHT TRIANGLES **Tell whether the given side lengths of a triangle can represent a right triangle.**

9. 9, 12, and 15 **right triangle**
10. 9, 10, and 15 **not a right triangle**
11. 36, 48, and 60 **right triangle**
12. 6, 10, and $2\sqrt{34}$ **right triangle**
13. 7, 14, and $7\sqrt{5}$ **right triangle**
14. 10, 12, and 20 **not a right triangle**

EXAMPLE 2 for Exs. 15–23

CLASSIFYING TRIANGLES **In Exercises 15–23, decide if the segment lengths form a triangle. If so, would the triangle be *acute, right,* or *obtuse?***

15. 10, 11, and 14 **triangle; acute**
16. 10, 15, and $5\sqrt{13}$ **triangle; right**
17. 24, 30, and $6\sqrt{43}$ **triangle; obtuse**
18. 5, 6, and 7 **triangle; acute**
19. 12, 16, and 20 **triangle; right**
20. 8, 10, and 12 **triangle; acute**
21. 15, 20, and 36 **not a triangle**
22. 6, 8, and 10 **triangle; right**
23. 8.2, 4.1, and 12.2 **triangle; obtuse**

24. ★ MULTIPLE CHOICE Which side lengths do not form a right triangle? **B**

Ⓐ 5, 12, 13 Ⓑ 10, 24, 28 Ⓒ 15, 36, 39 Ⓓ 50, 120, 130

25. ★ MULTIPLE CHOICE What type of triangle has side lengths of 4, 7, and 9? **C**

Ⓐ Acute scalene Ⓑ Right scalene
Ⓒ Obtuse scalene Ⓓ None of the above

B

26. ERROR ANALYSIS A student tells you that if you double all the sides of a right triangle, the new triangle is obtuse. *Explain* why this statement is incorrect.

26. *Sample answer:* If a triangle with sides *a*, *b*, and *c* is a right triangle with $a^2 + b^2 = c^2$, then a triangle with double side lengths would be $2a$, $2b$, and $2c$, and $(2a)^2 + (2b)^2 = (2c)^2$, $4a^2 + 4b^2 = 4c^2$, simplify to $a^2 + b^2 = c^2$, so if the sides of a right triangle are doubled, the new triangle is also a right triangle.

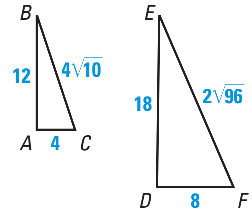
GRAPHING TRIANGLES **Graph points *A*, *B*, and *C*. Connect the points to form $\triangle ABC$. Decide whether $\triangle ABC$ is *acute, right,* or *obtuse.*** **27, 28. See margin for art.**

27. $A(-2, 4)$, $B(6, 0)$, $C(-5, -2)$ **right**
28. $A(0, 2)$, $B(5, 1)$, $C(1, -1)$ **acute**

2. The relationship of the longest side squared to the sum of the squares of the other two sides determines the angle classification. If the longest side squared is greater than the sum of the other two sides, then the triangle is obtuse; if they are equal, the triangle is a right triangle; if the longest side squared is less than the sum of the square of the other two sides, then the triangle is acute.

29. xy **ALGEBRA** Tell whether a triangle with side lengths $5x$, $12x$, and $13x$ (where $x > 0$) is *acute*, *right*, or *obtuse*. **right**

USING DIAGRAMS In Exercises 30 and 31, copy and complete the statement with <, >, or =, if possible. If it is not possible, *explain* why.



30. $m\angle A$ __?__ $m\angle D$ **>**

31. $m\angle B + m\angle C$ __?__ $m\angle E + m\angle F$ **<**

C 32. ★ **OPEN-ENDED MATH** The side lengths of a triangle are 6, 8, and x (where $x > 0$). What are the values of x that make the triangle a right triangle? an acute triangle? an obtuse triangle?
$2\sqrt{7}$ and 10; $2\sqrt{7} < x < 10$; $2 < x < 2\sqrt{7}$ or $10 < x < 14$

33. xy **ALGEBRA** The sides of a triangle have lengths x, $x + 4$, and 20. If the length of the longest side is 20, what values of x make the triangle acute? **$12 < x < 16$**

34. **CHALLENGE** The sides of a triangle have lengths $4x + 6$, $2x + 1$, and $6x - 1$. If the length of the longest side is $6x - 1$, what values of x make the triangle obtuse? **$x > 4.5$**

PROBLEM SOLVING

EXAMPLE 3 A
for Ex. 35

35. **PAINTING** You are making a canvas frame for a painting using stretcher bars. The rectangular painting will be 10 inches long and 8 inches wide. Using a ruler, how can you be certain that the corners of the frame are 90°?

Measure diagonally across the painting and it should be about 12.8 inches.

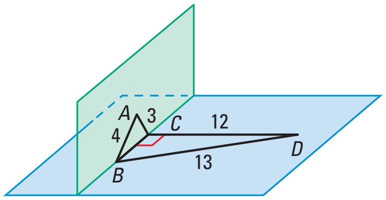
36. **WALKING** You walk 749 feet due east to the gym from your home. From the gym you walk 800 feet southwest to the library. Finally, you walk 305 feet from the library back home. Do you live directly north of the library? *Explain.* **no; $749^2 + 305^2 \neq 800^2$**

37. **MULTI-STEP PROBLEM** Use the diagram shown.

a. Find BC. **5**

b. Use the Converse of the Pythagorean Theorem to show that $\triangle ABC$ is a right triangle.

c. Draw and label a similar diagram where $\triangle DBC$ remains a right triangle, but $\triangle ABC$ is not. **See margin.**



37b. $3^2 + 4^2 = 5^2$ therefore $\triangle ABC$ is a right triangle.

Avoiding Common Errors

Exercise 6 When evaluating $(4\sqrt{19})^2$, some students may calculate it as $16 + 19$ or as $4 \cdot 19$. Review that $(4\sqrt{19})^2$ means $4\sqrt{19} \cdot 4\sqrt{19}$, or $16 \cdot 19$, which is 304.

Study Strategy

Exercises 27–28 Encourage students to check their answers by drawing the triangles on graph paper and using a protractor.

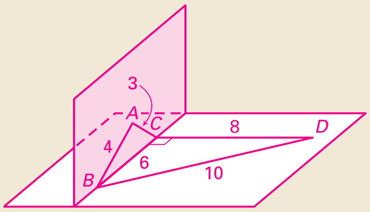
Mathematical Reasoning

Exercise 32 Have students think about whether x needs to be the length of the longest side. Students should realize that to solve this problem they must break it into two cases: (1) $6^2 + 8^2 = x^2$ and (2) $6^2 + x^2 = 8^2$.

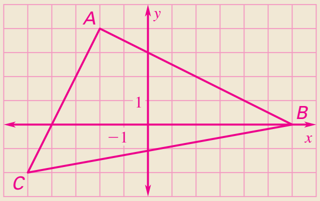
Avoiding Common Errors

Exercise 33 Students may forget the middle term when they evaluate $(x + 4)^2$. Review that $(x + 4)(x + 4) = x^2 + 8x + 16$.

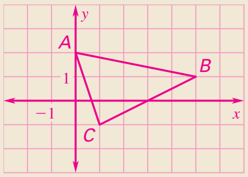
37c. *Sample:*



27.



28.



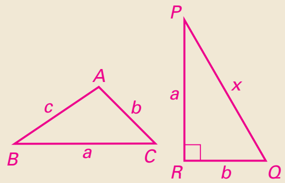
Internet Reference

Exercise 38 Additional information about volleyball can be found at www.volleyball.org/playing/index.html

Teaching Strategy

Exercise 40 Review all the theorems involving inequalities that students have learned in earlier chapters of this course, especially the Triangle Inequality Theorem, the Hinge Theorem, and the Converse of the Hinge Theorem.

41. Given: In $\triangle ABC$, $c^2 > a^2 + b^2$ where c is the length of the longest side.
Prove: $\triangle ABC$ is obtuse.

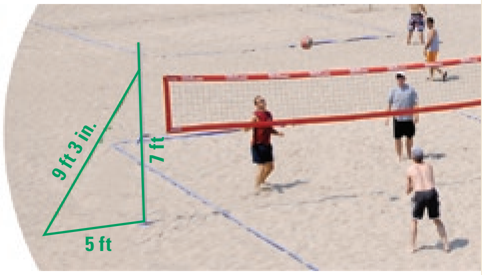


Statements (Reasons)

1. In $\triangle ABC$, $c^2 > a^2 + b^2$ where c is the length of the longest side. In $\triangle PQR$, $\angle R$ is a right angle. (Given)
2. $a^2 + b^2 = x^2$ (Pythagorean Theorem)
3. $c^2 > x^2$ (Substitution)
4. $c > x$ (A property of square roots)
5. $m\angle R = 90°$ (Definition of a right angle)
6. $m\angle C > m\angle R$ (Converse of the Hinge Theorem)
7. $m\angle C > 90°$ (Substitution Property of Equality)
8. $\angle C$ is an obtuse angle. (Definition of an obtuse angle)
9. $\triangle ABC$ is an obtuse triangle. (Definition of an obtuse triangle)

38. No; $7^2 + 5^2 \neq \left(\frac{37}{4}\right)^2$. *Sample answer:* Use about $9\frac{1}{2}$ feet of rope instead of $9\frac{1}{4}$ feet.

38. ★ SHORT RESPONSE You are setting up a volleyball net. To stabilize the pole, you tie one end of a rope to the pole 7 feet from the ground. You tie the other end of the rope to a stake that is 5 feet from the pole. The rope between the pole and stake is about 9 feet 3 inches long. Is the pole perpendicular to the ground? *Explain.* If it is not, how can you fix it?



B **39. ★ EXTENDED RESPONSE** You are considering buying a used car. You would like to know whether the frame is sound. A sound frame of the car should be rectangular, so it has four right angles. You plan to measure the shadow of the car on the ground as the sun shines directly on the car.

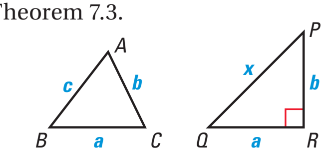
a. You make a triangle with three tape measures on one corner. It has side lengths 12 inches, 16 inches, and 20 inches. Is this a right triangle? *Explain.* **yes; $12^2 + 16^2 = 20^2$**

b. You make a triangle on a second corner with side lengths 9 inches, 12 inches, and 18 inches. Is this a right triangle? *Explain.* **no; $9^2 + 12^2 \neq 18^2$**

c. The car owner says the car was never in an accident. Do you believe this claim? *Explain.* **No; if the car was not in an accident, the angles should form a right triangle.**

40. PROVING THEOREM 7.3 Copy and complete the proof of Theorem 7.3.

GIVEN ▶ In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side.
PROVE ▶ $\triangle ABC$ is an acute triangle.



Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x, where $\angle R$ is a right angle and x is the length of the longest side. Compare lengths c and x.

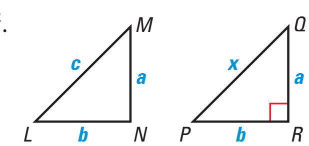
STATEMENTS	REASONS
1. In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side. In $\triangle PQR$, $\angle R$ is a right angle.	1. _?_ **Given**
2. $a^2 + b^2 = x^2$	2. _?_ **Pythagorean Theorem**
3. $c^2 < x^2$	3. _?_ **Substitution Property**
4. $c < x$	4. A property of square roots
5. $m\angle R = 90°$	5. _?_ **Definition of a right angle**
6. $m\angle C < m\angle$ _?_ **R**	6. Converse of the Hinge Theorem
7. $m\angle C < 90°$	7. _?_ **Substitution Property**
8. $\angle C$ is an acute angle.	8. _?_ **Definition of acute angle**
9. $\triangle ABC$ is an acute triangle.	9. _?_ **Definition of acute triangle**

41. PROVING THEOREM 7.4 Prove Theorem 7.4. Include a diagram and GIVEN and PROVE statements. (*Hint:* Look back at Exercise 40.) **See margin.**

42. PROVING THEOREM 7.2 Prove the Converse of the Pythagorean Theorem. **See margin.**

GIVEN ▶ In $\triangle LMN$, $\overline{LM}$ is the longest side, and $c^2 = a^2 + b^2$.
PROVE ▶ $\triangle LMN$ is a right triangle.

Plan for Proof Draw right $\triangle PQR$ with side lengths a, b, and x. Compare lengths c and x.



★ = **STANDARDIZED TEST PRACTICE**

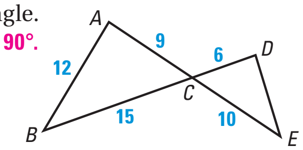
42. Statements (Reasons)

1. In $\triangle LMN$, $\overline{LM}$ is the longest side, and $c^2 = a^2 + b^2$. In $\triangle PQR$, $\angle R$ is a right angle. (Given)
2. $a^2 + b^2 = x^2$ (Pythagorean Theorem)
3. $c^2 = x^2$ (Substitution Property of Equality)
4. $c = x$ (A property of square roots)
5. $\triangle LMN \cong \triangle PQR$ (SSS Congruence Postulate)
6. $\angle N \cong \angle R$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
7. $m\angle N = 90°$ (Definition of congruent angles)
8. $\triangle LMN$ is a right triangle. (Definition of a right triangle)

44a, 44c. See Additional Answers.

44b. $\triangle ABC$ is not a right triangle; $\overline{AB}$ is the longest side, so $\angle C$ would have to be the right angle, but the slopes of $\overline{AC}$ and $\overline{BC}$ are not opposite reciprocals, so the line segments are not perpendicular and therefore, there is no right angle.

43. ★ **SHORT RESPONSE** *Explain* why $\angle D$ must be a right angle.
$\triangle ABC \sim \triangle DEC$, $\angle BAC$ is 90°, so $\angle EDC$ must also be 90°.



44. **COORDINATE PLANE** Use graph paper.

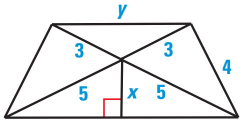
a. Graph $\triangle ABC$ with $A(-7, 2)$, $B(0, 1)$ and $C(-4, 4)$. See margin.

b. Use the slopes of the sides of $\triangle ABC$ to determine whether it is a right triangle. *Explain.*

c. Use the lengths of the sides of $\triangle ABC$ to determine whether it is a right triangle. *Explain.* See margin.

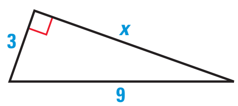
d. Did you get the same answer in parts (b) and (c)? If not, *explain* why. yes

45. **C** **CHALLENGE** Find the values of x and y.
$x = \sqrt{5}$, $y = \frac{12}{5}\sqrt{5}$

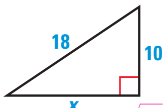


QUIZ

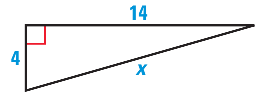
Find the unknown side length. Write your answer in simplest radical form.

1.


$6\sqrt{2}$

2.


$4\sqrt{14}$

3.


$2\sqrt{53}$

Classify the triangle formed by the side lengths as *acute, right,* or *obtuse.*

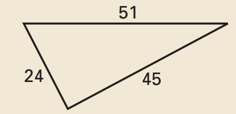
4. 6, 7, and 9 acute
5. 10, 12, and 16 obtuse
6. 8, 16, and $8\sqrt{6}$ obtuse
7. 20, 21, and 29 right
8. 8, 3, $\sqrt{73}$ right
9. 8, 10, and 12 acute

5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

1. Tell whether the triangle is a right triangle. yes

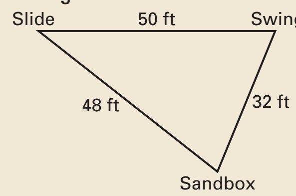


Decide if the segment lengths can form a triangle. If so, tell whether the triangle is acute, right, or obtuse.

2. 7, 11, $3\sqrt{17}$ yes; acute because $153 < 170$

3. 4.1, 9.2, 5.6 yes; obtuse because $84.64 > 48.17$

4. A playground has a slide, a swing, and a sandbox. The slide and the swing are 50 feet apart, the swing and sandbox are 32 feet apart, and the slide and sandbox are 48 feet apart. Do the three pieces of apparatus form a right triangle? no



Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

1 PLAN AND PREPARE

Explore the Concept
- Students will explore geometric means in right triangles.
- This activity leads into studying similar right triangles in this lesson.

Materials
Each student will need:
- straight edge
- scissors

Recommended Time
Work activity: 10 min

Discuss results: 5 min

Grouping
Students should work individually.

2 TEACH

Tips for Success
Before cutting apart the triangles in Step 3, be sure students label all the angles.

Key Question
- In Step 4, is $\angle 3 \cong \angle 6 \cong \angle 9$? Why? **Yes, each is complementary to $\angle 7$.**

Alternative Strategy
Start with a single right triangle and cut it along the altitude to the hypotenuse. Show that the two triangles are similar. Then show that each is similar to the original triangle.

Key Discovery
The altitude to the hypotenuse forms two triangles, similar to each other and to the original.

3 ASSESS AND RETEACH

1. If you drew the altitudes from the right angles in each of the triangles in Step 3 and then cut along those altitudes, how would the six new triangles be related? **They would all be similar to each other.**

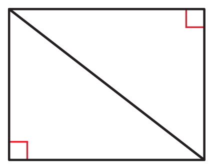
Similar Right Triangles

MATERIALS • rectangular piece of paper • ruler • scissors • colored pencils

QUESTION How are geometric means related to the altitude of a right triangle?

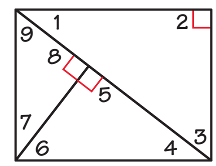
EXPLORE Compare right triangles

STEP 1



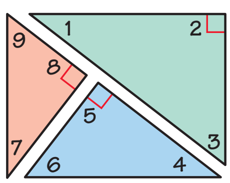
Draw a diagonal Draw a diagonal on your rectangular piece of paper to form two congruent right triangles.

STEP 2



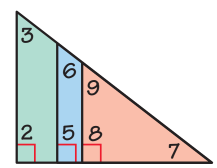
Draw an altitude Fold the paper to make an altitude to the hypotenuse of one of the triangles.

STEP 3



Cut and label triangles Cut the rectangle into the three right triangles that you drew. Label the angles and color the triangles as shown.

STEP 4

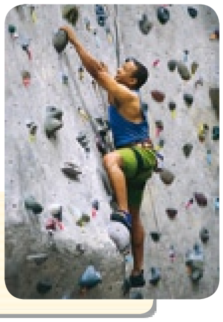


Arrange the triangles Arrange the triangles so $\angle 1$, $\angle 4$, and $\angle 7$ are on top of each other as shown.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. How are the two smaller right triangles related to the large triangle? **They are similar to it.**
2. *Explain* how you would show that the green triangle is similar to the red triangle. **Show that corresponding sides have a constant proportion.**
3. *Explain* how you would show that the red triangle is similar to the blue triangle. **Show that corresponding sides have a constant proportion.**
4. The *geometric mean* of a and b is x if $\frac{a}{x} = \frac{x}{b}$. Write a proportion involving the side lengths of two of your triangles so that one side length is the geometric mean of the other two lengths in the proportion. **Check students' work.**

7.3 Use Similar Right Triangles



Before You identified the altitudes of a triangle.

Now You will use properties of the altitude of a right triangle.

Why? So you can determine the height of a wall, as in Example 4.

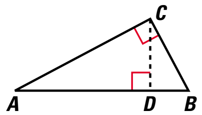
Key Vocabulary
- altitude of a triangle
- geometric mean
- similar polygons

When the altitude is drawn to the hypotenuse of a right triangle, the two smaller triangles are similar to the original triangle and to each other.

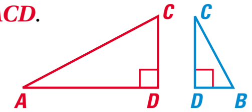
THEOREM *For Your Notebook*

THEOREM 7.5

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.

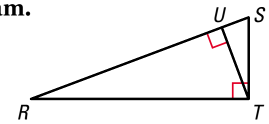


Plan for Proof of Theorem 7.5 First prove that $\triangle CBD \sim \triangle ABC$. Each triangle has a right angle and each triangle includes $\angle B$. The triangles are similar by the AA Similarity Postulate. Use similar reasoning to show that $\triangle ACD \sim \triangle ABC$.

To show $\angle CBD \sim \triangle ACD$, begin by showing $\angle ACD \cong \angle B$ because they are both complementary to $\angle DCB$. Each triangle also has a right angle, so you can use the AA Similarity Postulate.

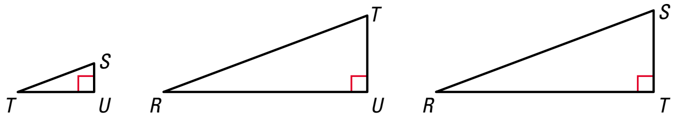
EXAMPLE 1 **Identify similar triangles**

Identify the similar triangles in the diagram.



Solution

Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.



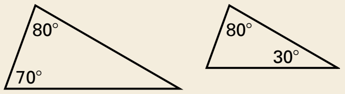
▶ $\triangle TSU \sim \triangle RTU \sim \triangle RST$

1 PLAN AND PREPARE

Warm-Up Exercises

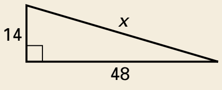
Also available online

1. Are these triangles similar? If so, give the reason.



Yes; the AA Similarity Postulate

2. Find x. **50**



Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 0.5 block with previous lesson
0.5 block with next lesson

- See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How can you find the length of the altitude to the hypotenuse of a right triangle? **Tell students they will learn how to answer this question by using properties of the altitude of a right triangle.**

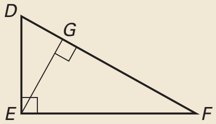
Motivating the Lesson

A ladder rests against a building. The length of a piece of rope from the base of the building and perpendicular to the ladder is related to two other distances: from the base of the ladder to the building, and from the base of the building to where the ladder touches the building. In this lesson students will explore that relationship.

3 TEACH

Extra Example 1

Identify the similar triangles in the diagram.



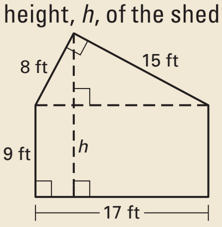
$\triangle DEF \sim \triangle DGE \sim \triangle EGF$

Key Question to Ask for Example 1

- How would you use the AA Similarity Postulate to show the triangles are similar? **The three triangles all have right angles. $\triangle RUT$ and $\triangle RTS$ both have acute $\angle R$, and $\triangle RTS$ and $\triangle STU$ both have acute $\angle S$. So the triangles are similar by the AA Similarity Postulate.**

Extra Example 2

The figure shows the side view of a tool shed. What is the maximum height, *h*, of the shed? **16.1 ft**

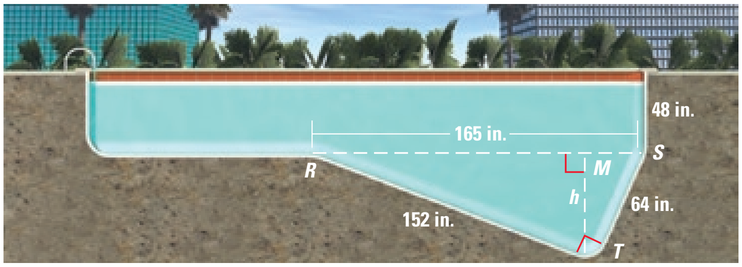


AVOID ERRORS

Notice that if you tried to write a proportion using $\triangle RTM$ and $\triangle TSM$, there would be two unknowns, so you would not be able to solve for *h*.

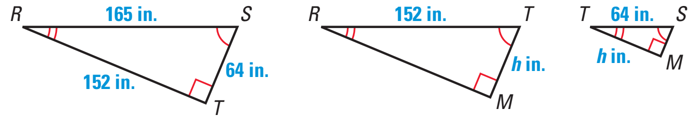
EXAMPLE 2 Find the length of the altitude to the hypotenuse

SWIMMING POOL The diagram below shows a cross-section of a swimming pool. What is the maximum depth of the pool?



Solution

STEP 1 **Identify** the similar triangles and sketch them.



$\triangle RST \sim \triangle RTM \sim \triangle TSM$

STEP 2 **Find** the value of h. Use the fact that $\triangle RST \sim \triangle RTM$ to write a proportion.

$\frac{TM}{ST} = \frac{TR}{SR}$ **Corresponding side lengths of similar triangles are in proportion.**

$\frac{h}{64} = \frac{152}{165}$ **Substitute.**

$165h = 64(152)$ **Cross Products Property**

$h \approx 59$ **Solve for h.**

STEP 3 **Read** the diagram above. You can see that the maximum depth of the pool is $h + 48$, which is about $59 + 48 = 107$ inches.

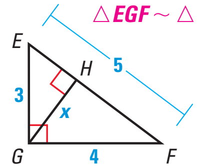
▶ The maximum depth of the pool is about 107 inches.

Animated Geometry at my.hrw.com

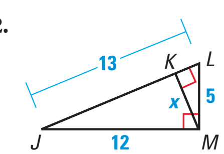
✓ GUIDED PRACTICE for Examples 1 and 2

Identify the similar triangles. Then find the value of *x*.

1. $\triangle EGF \sim \triangle GHF \sim \triangle EHG$; $\frac{12}{5}$

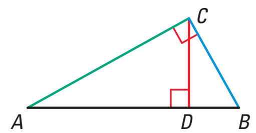


2.

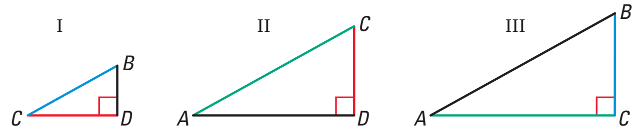


$\triangle LMJ \sim \triangle MKJ \sim \triangle LKM$; $\frac{60}{13}$

GEOMETRIC MEANS You have learned that the *geometric mean* of two numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. Consider right $\triangle ABC$. From Theorem 7.5, you know that altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.



READ SYMBOLS
Remember that an altitude is defined as a segment. So, $\overline{CD}$ refers to an altitude in $\triangle ABC$ and CD refers to its length.



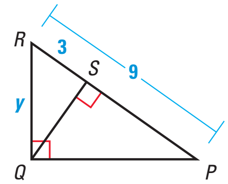
Notice that $\overline{CD}$ is the longer leg of $\triangle CBD$ and the shorter leg of $\triangle ACD$. When you write a proportion comparing the leg lengths of $\triangle CBD$ and $\triangle ACD$, you can see that CD is the geometric mean of BD and AD. As you see below, CB and AC are also geometric means of segment lengths in the diagram.

Proportions Involving Geometric Means in Right $\triangle ABC$

length of shorter leg of I length of shorter leg of II	→	$\frac{BD}{CD} = \frac{CD}{AD}$	←	length of longer leg of I length of longer leg of II
length of hypotenuse of III length of hypotenuse of I	→	$\frac{AB}{CB} = \frac{CB}{DB}$	←	length of shorter leg of III length of shorter leg of I
length of hypotenuse of III length of hypotenuse of II	→	$\frac{AB}{AC} = \frac{AC}{AD}$	←	length of longer leg of III length of longer leg of II

EXAMPLE 3 Use a geometric mean

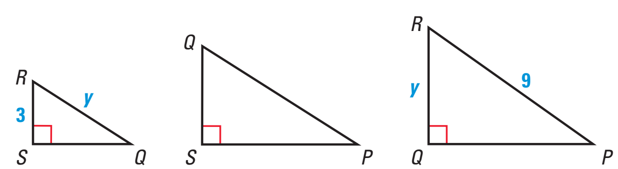
xy **Find the value of y. Write your answer in simplest radical form.**



Solution

REVIEW SIMILARITY
Notice that $\triangle RQS$ and $\triangle RPQ$ both contain the side with length y, so these are the similar triangles to use to solve for y.

STEP 1 **Draw** the three similar triangles.



STEP 2 **Write** a proportion.

$$\frac{\text{length of hyp. of } \triangle RPQ}{\text{length of hyp. of } \triangle RQS} = \frac{\text{length of shorter leg of } \triangle RPQ}{\text{length of shorter leg of } \triangle RQS}$$

$\frac{9}{y} = \frac{y}{3}$ **Substitute.**

$27 = y^2$ **Cross Products Property**

$\sqrt{27} = y$ **Take the positive square root of each side.**

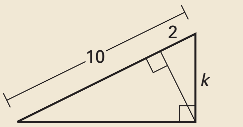
$3\sqrt{3} = y$ **Simplify.**

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

Extra Example 3

Find the value of k. $2\sqrt{5}$



Differentiated Instruction

Kinesthetic Learners Some students may have difficulty visualizing how to sketch similar triangles from a diagram. Have these students use tracing paper to trace each of the three triangles in the diagram in **Example 3**. Ask them to cut each triangle out. Have them arrange the triangles as they were in the diagram and then manipulate each one by rotating and/or flipping it to show all three similar triangles in the same orientation.

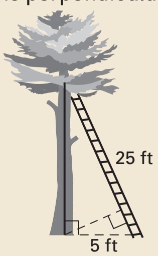
See also the *Differentiated Instruction Resources* for more strategies.

Reading Strategy

Theorem 6 Students can think of the proportion as "part of the hypotenuse is to the altitude as the altitude is to the other part of the hypotenuse."

Extra Example 4

You are standing by a tree as shown in the diagram. A 25 foot ladder is leaning against the tree. What is the length of a piece of rope that goes from the base of the tree and is perpendicular to the ladder? **4.9 ft**



Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How can you find the length of the altitude to the hypotenuse of a right triangle?

- **The altitude to the hypotenuse in a right triangle divides the triangle into 2 triangles, each similar to the other and each similar to the original triangle.**
- **The altitude to the hypotenuse in a right triangle is the geometric mean of the two parts of the hypotenuse.**

Write and solve the proportion $\frac{\text{part of hyp.}}{\text{altitude}} = \frac{\text{altitude}}{\text{other part of hyp.}}$

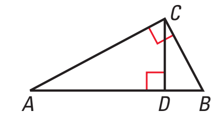
WRITE PROOFS
In Exercise 32, you will use the geometric mean theorems to prove the Pythagorean Theorem.

THEOREMS — *For Your Notebook*

THEOREM 7.6 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

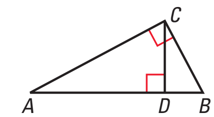


$\frac{BD}{CD} = \frac{CD}{AD}$

THEOREM 7.7 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

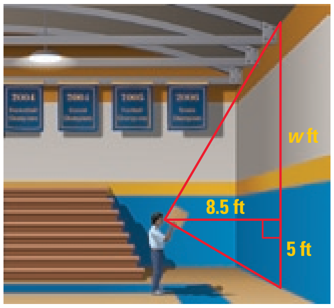


$\frac{AB}{CB} = \frac{CB}{DB}$ and $\frac{AB}{AC} = \frac{AC}{AD}$

EXAMPLE 4 Find a height using indirect measurement

ROCK CLIMBING WALL To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall.

You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye and the distance from you to the gym wall. Approximate the height of the gym wall.



Solution

By the Geometric Mean (Altitude) Theorem, you know that 8.5 is the geometric mean of w and 5.

$\frac{w}{8.5} = \frac{8.5}{5}$ **Write a proportion.**

$w \approx 14.5$ **Solve for w.**

▶ So, the height of the wall is $5 + w \approx 5 + 14.5 = 19.5$ feet.

✓ GUIDED PRACTICE for Examples 3 and 4

3. In Example 3, which theorem did you use to solve for y? *Explain.*

3. The Geometric Mean (Leg) Theorem; set the ratios of the hypotenuse of the large triangle to the shorter leg and the hypotenuse of the small triangle to the shorter leg equal to each other.

4. Mary is 5.5 feet tall. How far from the wall in Example 4 would she have to stand in order to measure its height? **about 8.77 ft**

Differentiated Instruction

Below Level Encourage students to rewrite the Geometric Mean Theorems in the form (altitude)^2 = (part of hypotenuse) × (other part of hypotenuse) and (leg)^2 = (hypotenuse) × (part of hypotenuse closer to leg).

See also the *Differentiated Instruction Resources* for more strategies.

7.3 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 15, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 20, 31, and 34

SKILL PRACTICE

A

1. **VOCABULARY** Copy and complete: Two triangles are _?_ if their corresponding angles are congruent and their corresponding side lengths are proportional. **similar**

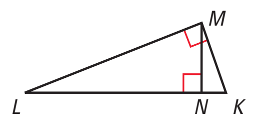
2. ★ **WRITING** In your own words, explain *geometric mean*. **See margin.**

EXAMPLE 1 for Exs. 3–4

IDENTIFYING SIMILAR TRIANGLES Identify the three similar right triangles in the given diagram.

3. 

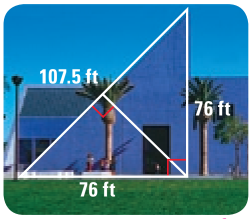
△FHG ~ △HEG ~ △FEH

4. 

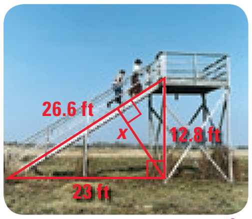
△KML ~ △MNL ~ △KNM

EXAMPLE 2 for Exs. 5–7

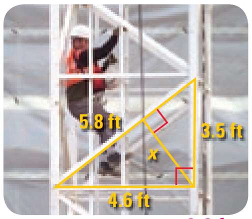
FINDING ALTITUDES Find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.

5. 

53.7 ft

6. 

11.1 ft

7. 

2.8 ft

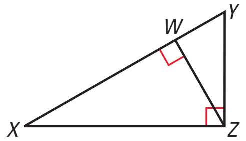
EXAMPLES 3 and 4 for Exs. 8–18

COMPLETING PROPORTIONS Write a similarity statement for the three similar triangles in the diagram. Then complete the proportion.

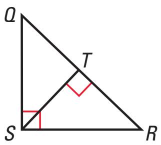
8. $\frac{XW}{?} = \frac{ZW}{YW}$

9. $\frac{?}{SQ} = \frac{SQ}{TQ}$

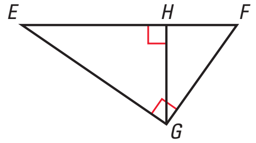
10. $\frac{EF}{EG} = \frac{EG}{?}$



△YZX ~ △ZWX ~ △YWZ; ZW

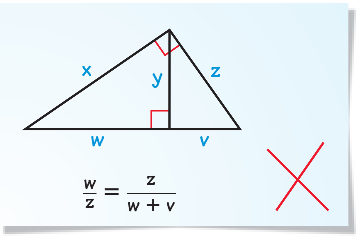


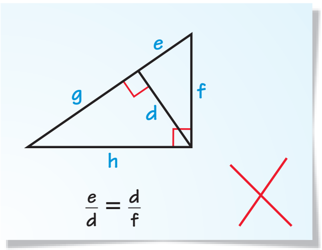
△QSR ~ △STR ~ △QTS; RQ



△GEF ~ △HEG ~ △HGF; EH

ERROR ANALYSIS *Describe* and correct the error in writing a proportion for the given diagram.

11. 

12. 

11. ***Sample answer:*** **The proportion must compare corresponding parts; $\frac{v}{z} = \frac{z}{w+v}$.**

12. When using the altitude and parts of the hypotenuse, you must use both pieces of the large triangle's hypotenuse, $\frac{e}{d} = \frac{d}{g}$.

2. ***Sample answer:*** **If a proportion is formed such that the same value is in the numerator of one fraction and in the denominator of the other fraction, this value is the geometric mean of the other two numbers in the proportion.**

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1: Exs. 5, 6, 13–16
Exs. 1–7, 13–15
Day 2:
Exs. 8–12, 16–21, 29–33

Average:
Day 1:
Exs. 1–7, 13–15, 19, 20
Day 2:
Exs. 8–12, 16–18, 22–26, 30–37

Advanced:
Day 1:
Exs. 1–7, 13–15, 19, 20, 28*
Day 2:
Exs. 9, 10, 17, 18, 21–27, 31–38*

Block:
Exs. 1–7, 13–15, 19, 20
(with previous lesson)
Exs. 8–12, 16–18, 22–26, 30–37
(with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 10, 30, 31
Average: 4, 6, 16, 31, 33
Advanced: 4, 7, 18, 31, 34

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

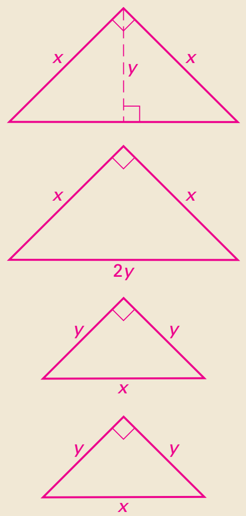
Avoiding Common Errors

Exercises 9–10 Students may use the wrong part of the hypotenuse in their proportion. Have them redraw the three triangles side-by-side so corresponding angles are in the same relative positions before they write the similarity statements. If they draw the triangles so they do not look isosceles, it will be easier for them to tell which is the longer leg in each triangle.

Teaching Strategy

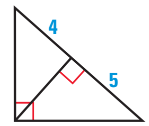
Exercises 13–18 Encourage students to start writing the proportions by filling in the geometric means, such as $\frac{\ }{x} = \frac{x}{\ }$ for Exercise 13 and $\frac{\ }{18} = \frac{18}{\ }$ for Exercise 14. Then they can fill in the proportion and solve it.

28.



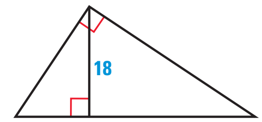
FINDING LENGTHS **Find the value of the variable. Round decimal answers to the nearest tenth.**

13.



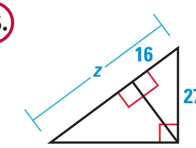
about 6.7

14.



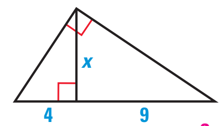
27

15.



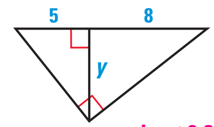
about 45.6

16.



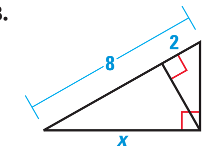
6

17.



about 6.3

18.

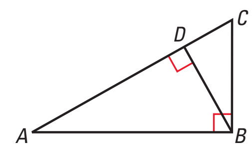


about 6.9

B **19.** ★ **MULTIPLE CHOICE** Use the diagram at the right. Decide which proportion is false. **C**

Ⓐ $\frac{DB}{DC} = \frac{DA}{DB}$ Ⓑ $\frac{CA}{AB} = \frac{AB}{AD}$

Ⓒ $\frac{CA}{BA} = \frac{BA}{CA}$ Ⓓ $\frac{DC}{BC} = \frac{BC}{CA}$

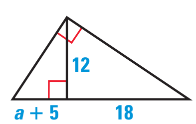


20. ★ **MULTIPLE CHOICE** In the diagram in Exercise 19 above, $AC = 36$ and $BC = 18$. Find AD. If necessary, round to the nearest tenth. **C**

Ⓐ 9 Ⓑ 15.6 Ⓒ 27 Ⓓ 31.2

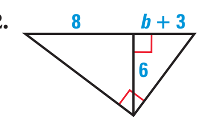
xy **ALGEBRA** **Find the value(s) of the variable(s).**

21.



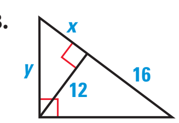
3

22.



1.5

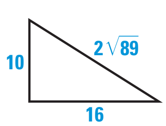
23.



$x = 9$, $y = 15$, $z = 20$

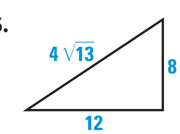
USING THEOREMS **Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answers to the nearest tenth.**

24.



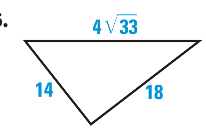
right triangle; about 8.5

25.



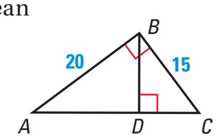
right triangle; about 6.7

26.



not a right triangle

C **27.** **FINDING LENGTHS** Use the Geometric Mean Theorems to find AC and BD. **25, 12**



28. **CHALLENGE** Draw a right isosceles triangle and label the two leg lengths x. Then draw the altitude to the hypotenuse and label its length y. Now draw the three similar triangles and label any side length that is equal to either x or y. What can you conclude about the relationship between the two smaller triangles? *Explain.* **See margin for art.** ***Sample answer:*** **The two smaller triangles are congruent to each other and are also isosceles triangles.**

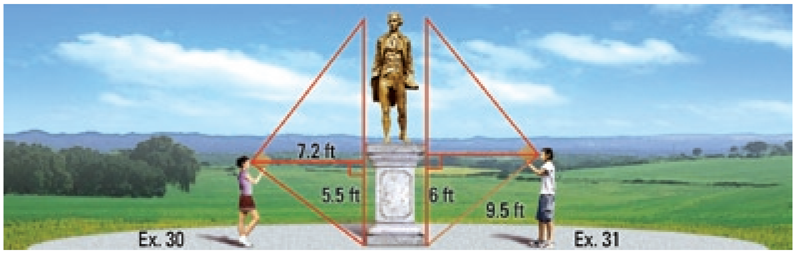
◯ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

A **29.** **DOGHOUSE** The peak of the doghouse shown forms a right angle. Use the given dimensions to find the height of the roof. **about 1.1 ft**

EXAMPLE 4
for Exs. 30–31

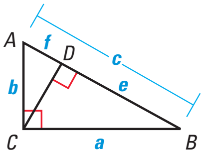
30. **MONUMENT** You want to determine the height of a monument at a local park. You use a cardboard square to line up the top and bottom of the monument. Mary measures the vertical distance from the ground to your eye and the distance from you to the monument. Approximate the height of the monument (as shown at the left below). **about 14.9 ft**



31. about 15 ft; no; the values are slightly off because the measurements are not exact.

31. ★ **SHORT RESPONSE** Paul is standing on the other side of the monument in Exercise 30 (as shown at the right above). He has a piece of rope staked at the base of the monument. He extends the rope to the cardboard square he is holding lined up to the top and bottom of the monument. Use the information in the diagram above to approximate the height of the monument. Do you get the same answer as in Exercise 30? *Explain.*

B **32.** **PROVING THE PYTHAGOREAN THEOREM** Use the diagram of $\triangle ABC$. Copy and complete the proof of the Pythagorean Theorem.



GIVEN ▶ In $\triangle ABC$, $\angle BCA$ is a right angle.

PROVE ▶ $c^2 = a^2 + b^2$

STATEMENTS	REASONS
1. Draw $\triangle ABC$. $\angle BCA$ is a right angle.	1. ? **Given**
2. Draw a perpendicular from C to $\overline{AB}$.	2. Perpendicular Postulate
3. $\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$	3. ? **Geometric Mean (leg) Theorem**
4. $ce = a^2$ and $cf = b^2$	4. ? **Cross Products Property**
5. $ce + b^2 =$? $+ b^2$ **a^2**	5. Addition Property of Equality
6. $ce + cf = a^2 + b^2$	6. ? **Substitution Property of Equality**
7. $c(e + f) = a^2 + b^2$	7. ? **Distributive Property**
8. $e + f =$? **c**	8. Segment Addition Postulate
9. $c \cdot c = a^2 + b^2$	9. ? **Substitution Property of Equality**
10. $c^2 = a^2 + b^2$	10. Simplify.

Avoiding Common Errors

Exercise 29 Some students may write $\frac{1.5}{x} = \frac{x}{1.5}$ as the proportion. Remind them that they must use the two parts of the hypotenuse, so the first step is to use the Pythagorean Theorem to find the complete hypotenuse. Then they can use the Pythagorean Theorem or Geometric Mean Theorem to find x.

Mathematical Reasoning

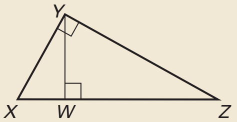
Exercise 32 To identify each Reason, students should try to determine what was done to each previous Statement to obtain the new Statement.

5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

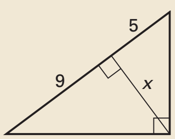
1. Identify the three similar right triangles in the diagram.



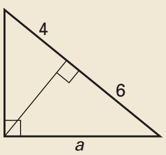
$\triangle XYZ \sim \triangle XWY \sim \triangle YWZ$

Find the value of the variable.

2. $3\sqrt{5}$



3. $2\sqrt{15}$



Online Quiz

Available at **my.hrw.com**

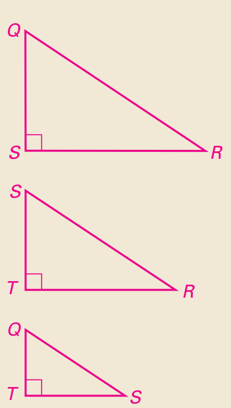
Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

34a.



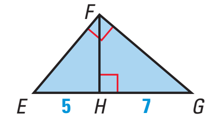
35–37. See Additional Answers.

33a. $\overline{FH}$, $\overline{GF}$, $\overline{EF}$; each segment has a vertex as an endpoint and is perpendicular to the opposite side.

34a. See margin for art; the right angles correspond and from there you find the shorter leg and the longer leg, keeping in mind which way you choose and being consistent with all three triangles.

34c. $\overline{RS}$. *Sample answer:* $\overline{RQ}$ is the hypotenuse of the large triangle and $\overline{RT}$ is the long leg of the medium triangle, so the relationship for the geometric mean requires a segment that is a hypotenuse and a long leg.

33. MULTI-STEP PROBLEM Use the diagram.

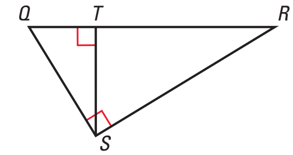


a. Name all the altitudes in $\triangle EGF$. *Explain.*

b. Find *FH*. $\sqrt{35}$

c. Find the area of the triangle. **about 35.5**

34. ★ **EXTENDED RESPONSE** Use the diagram.



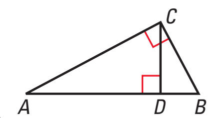
a. Sketch the three similar triangles in the diagram. Label the vertices. *Explain* how you know which vertices correspond.

b. Write similarity statements for the three triangles. $\triangle QSR \sim \triangle STR \sim \triangle QTS$

c. Which segment's length is the geometric mean of *RT* and *RQ*? *Explain* your reasoning.

PROVING THEOREMS **In Exercises 35–37, use the diagram and GIVEN statements below.** **35–37. See margin.**

GIVEN ▶ $\triangle ABC$ is a right triangle.
Altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.



35. Prove Theorem 7.5 by using the Plan for Proof.

36. Prove the Geometric Mean (Altitude) Theorem by showing $\frac{BD}{CD} = \frac{CD}{AD}$.

37. Prove Geometric Mean (Leg) Theorem by showing $\frac{AB}{CB} = \frac{CB}{DB}$ and $\frac{AB}{AC} = \frac{AC}{AD}$.

38. CHALLENGE The *harmonic mean* of *a* and *b* is $\frac{2ab}{a+b}$. The Greek mathematician Pythagoras found that three equally taut strings on stringed instruments will sound harmonious if the length of the middle string is equal to the harmonic mean of the lengths of the shortest and longest string.

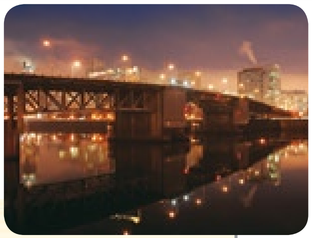
a. Find the harmonic mean of 10 and 15. **12**

b. Find the harmonic mean of 6 and 14. **8.4**

c. Will equally taut strings whose lengths have the ratio 4 : 6 : 12 sound harmonious? *Explain* your reasoning. **Yes; when you compute the harmonic mean using 4 and 12, you get 6.**

See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com

7.4 Special Right Triangles



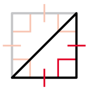
Before You found side lengths using the Pythagorean Theorem.

Now You will use the relationships among the sides in special right triangles.

Why? So you can find the height of a drawbridge, as in Ex. 28.

Key Vocabulary
- **isosceles triangle**

A 45°-45°-90° triangle is an *isosceles right triangle* that can be formed by cutting a square in half as shown.



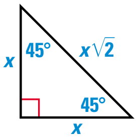
USE RATIOS
The extended ratio of the side lengths of a 45°-45°-90° triangle is $1:1:\sqrt{2}$.

THEOREM *For Your Notebook*

THEOREM 7.8 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg $\cdot \sqrt{2}$

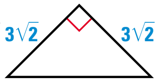


EXAMPLE 1 Find hypotenuse length in a 45°-45°-90° triangle

Find the length of the hypotenuse.

a.

b.



Solution

a. By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle, so by the 45°-45°-90° Triangle Theorem, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg $\cdot \sqrt{2}$ **45°-45°-90° Triangle Theorem**

$= 8\sqrt{2}$ **Substitute.**

b. By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg $\cdot \sqrt{2}$ **45°-45°-90° Triangle Theorem**

$= 3\sqrt{2} \cdot \sqrt{2}$ **Substitute.**

$= 3 \cdot 2$ **Product of square roots**

$= 6$ **Simplify.**

REVIEW ALGEBRA
Remember the following properties of radicals:
$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$
$\sqrt{a \cdot a} = a$
For a review of radical expressions, see SR6.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

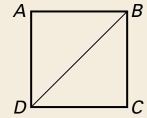
Simplify.

1. $6\sqrt{2} \cdot \sqrt{2}$ **12**

2. $\frac{6}{\sqrt{3}}$ **$2\sqrt{3}$**

3. $\frac{5}{\sqrt{2}}$ **$\frac{5\sqrt{2}}{2}$**

4. Find $m\angle DBC$ in square $ABCD$. **45°**



Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 0.5 block with previous lesson
0.5 block with next lesson
- See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How do you find the lengths of the sides of a 30°-60°-90° triangle and a 45°-45°-90° triangle? **Tell students they will learn how to answer this question by learning about two special right triangles.**

Motivating the Lesson

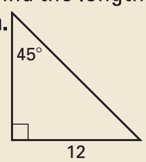
In an equilateral triangle, how does the length of an altitude compare to the length of a side? In a square, how does the length of a diagonal compare to the length of a side? In this lesson students will learn how to answer these questions by studying 45°-45°-90° triangles and 30°-60°-90° triangles.

3 TEACH

Extra Example 1

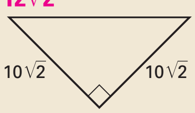
Find the length of the hypotenuse.

a.



$12\sqrt{2}$

b.



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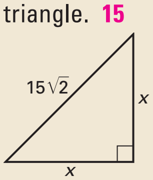
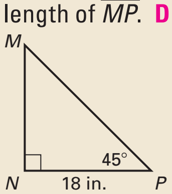
Extra Example 2

Find the length of the legs in the triangle. **15**



Extra Example 3

$\triangle MNP$ is a right triangle. Find the length of $\overline{MP}$. **D**

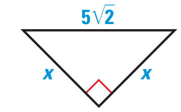


Ⓐ 18 in. Ⓑ $9\sqrt{2}$ in.

Ⓒ 36 in. Ⓓ $18\sqrt{2}$ in.

EXAMPLE 2 Find leg lengths in a 45°-45°-90° triangle

Find the lengths of the legs in the triangle.



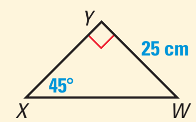
Solution

By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

hypotenuse $=$ leg $\cdot \sqrt{2}$	**45°-45°-90° Triangle Theorem**
$5\sqrt{2} = x \cdot \sqrt{2}$	**Substitute.**
$\frac{5\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$	**Divide each side by $\sqrt{2}$.**
$5 = x$	**Simplify.**

EXAMPLE 3 Standardized Test Practice

Triangle WXY is a right triangle. Find the length of $\overline{WX}$.



Ⓐ 50 cm Ⓑ $25\sqrt{2}$ cm

Ⓒ 25 cm Ⓓ $\frac{25\sqrt{2}}{2}$ cm

ELIMINATE CHOICES
You can eliminate choices C and D because the hypotenuse has to be longer than the leg.

Solution

By the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

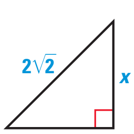
hypotenuse $=$ leg $\cdot \sqrt{2}$	**45°-45°-90° Triangle Theorem**
$WX = 25\sqrt{2}$	**Substitute.**

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Examples 1, 2, and 3

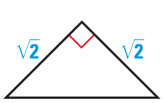
Find the value of the variable.

1.



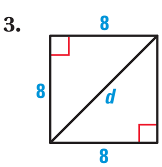
2

2.



2

3.



$8\sqrt{2}$

4. Find the leg length of a 45°-45°-90° triangle with a hypotenuse length of 6. $3\sqrt{2}$

Differentiated Instruction

Auditory Learners Have students work with a partner. As one student reads the 30°-60°-90° Triangle Theorem aloud, his or her partner should sketch a triangle to show the length of each leg and the hypotenuse, and also the measure of each angle. Have them change roles so each student has a chance to make a sketch. Have them repeat the process with the 45°-45°-90° Triangle Theorem.

See also the *Differentiated Instruction Resources* for more strategies.

A 30°-60°-90° triangle can be formed by dividing an equilateral triangle in half.

USE RATIOS
The extended ratio of the side lengths of a 30°-60°-90° triangle is $1 : \sqrt{3} : 2$.

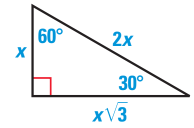
THEOREM — *For Your Notebook*

THEOREM 7.9 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$



EXAMPLE 4 Find the height of an equilateral triangle

LOGO The logo on the recycling bin at the right resembles an equilateral triangle with side lengths of 6 centimeters. What is the approximate height of the logo?

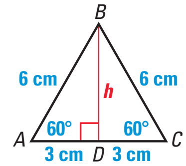
Solution

Draw the equilateral triangle described. Its altitude forms the longer leg of two 30°-60°-90° triangles. The length h of the altitude is approximately the height of the logo.

REVIEW MEDIAN
Remember that in an equilateral triangle, the altitude to a side is also the median to that side. So, altitude $\overline{BD}$ bisects $\overline{AC}$.

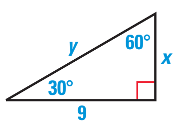
longer leg = shorter leg • $\sqrt{3}$

$h = 3 \cdot \sqrt{3} \approx 5.2$ cm



EXAMPLE 5 Find lengths in a 30°-60°-90° triangle

Find the values of x and y. Write your answer in simplest radical form.



STEP 1 **Find** the value of x.

longer leg = shorter leg • $\sqrt{3}$	30°-60°-90° Triangle Theorem
$9 = x\sqrt{3}$	Substitute.
$\frac{9}{\sqrt{3}} = x$	Divide each side by $\sqrt{3}$.
$\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$	Multiply numerator and denominator by $\sqrt{3}$.
$\frac{9\sqrt{3}}{3} = x$	Multiply fractions.
$3\sqrt{3} = x$	Simplify.

STEP 2 **Find** the value of y.

hypotenuse = 2 • shorter leg	30°-60°-90° Triangle Theorem
$y = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$	Substitute and simplify.

Key Question to Ask for Example 3

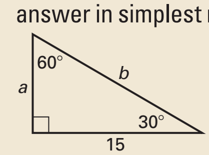
- When do you multiply by $\sqrt{2}$? When do you divide by $\sqrt{2}$? **You multiply by $\sqrt{2}$ when you are given the length of one leg of a 45°-45°-90° triangle and you are asked to find the length of the hypotenuse. You divide by $\sqrt{2}$ when you are given the hypotenuse and are asked to find the leg.**

Extra Example 4

Alex has a team logo patch in the shape of an equilateral triangle. If the sides of the patch are $2\frac{1}{2}$ inches long, what is the approximate height of the patch? **2.2 in.**

Extra Example 5

Find the values of a and b. Write your answer in simplest radical form.



$5\sqrt{3}$, $10\sqrt{3}$

Key Question to Ask for Example 5

- Which is the shorter leg in a 30°-60°-90° triangle? How do you know? **The shorter leg is opposite the smaller angle. *Sample explanation:* Theorem 5.11, page 328: In a triangle, the side opposite a larger angle is longer than the side opposite a smaller angle.**

Differentiated Instruction

Below Level In Example 5, students have to solve for x in the equation $9 = x\sqrt{3}$. Some students may find this method easier: Multiply both sides by $\sqrt{3}$ to get $9\sqrt{3} = 3x$, then divide both sides by 3 to get $3\sqrt{3} = x$.

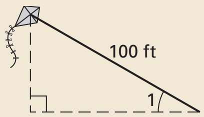
See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 6

A kite is attached to a 100 foot string as shown in the diagram. How far above the ground is the kite when the string forms the given angle?

a. $m\angle 1 = 45°$ **$50\sqrt{2} \approx 70.71$ ft**

b. $m\angle 1 = 30°$ **50 ft**



Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 6**. This activity is also part of **Power Presentations**.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you find the lengths of the sides of a 30°-60°-90° triangle and a 45°-45°-90° triangle?

- **In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of each leg.**
- **In a 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.**

In a 30°-60°-90° triangle the ratio of the sides is $x : x\sqrt{3} : 2x$. In a 45°-45°-90° triangle the ratio of the sides is $x : x : x\sqrt{2}$.

REWRITE MEASURES

To write 9.9 ft in feet and inches, multiply the decimal part by 12.

$12 \cdot 0.9 = 10.8$

So, 9.9 ft is about 9 feet 11 inches.

EXAMPLE 6 Find a height

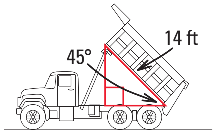
DUMP TRUCK The body of a dump truck is raised to empty a load of sand. How high is the 14 foot body from the frame when it is tipped upward at the given angle?

a. 45° angle **b.** 60° angle

Solution

a. When the body is raised 45° above the frame, the height h is the length of a leg of a 45°-45°-90° triangle. The length of the hypotenuse is 14 feet.

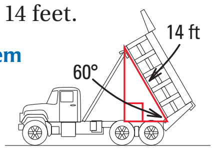
$14 = h \cdot \sqrt{2}$	45°-45°-90° Triangle Theorem
$\frac{14}{\sqrt{2}} = h$	Divide each side by $\sqrt{2}$.
$9.9 \approx h$	Use a calculator to approximate.



▶ When the angle of elevation is 45°, the body is about 9 feet 11 inches above the frame.

b. When the body is raised 60°, the height h is the length of the longer leg of a 30°-60°-90° triangle. The length of the hypotenuse is 14 feet.

hypotenuse $= 2 \cdot$ shorter leg	30°-60°-90° Triangle Theorem
$14 = 2 \cdot s$	Substitute.
$7 = s$	Divide each side by 2.
longer leg $=$ shorter leg $\cdot \sqrt{3}$	30°-60°-90° Triangle Theorem
$h = 7\sqrt{3}$	Substitute.
$h \approx 12.1$	Use a calculator to approximate.



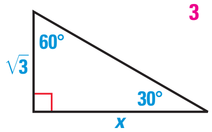
▶ When the angle of elevation is 60°, the body is about 12 feet 1 inch above the frame.

Animated Geometry at my.hrw.com

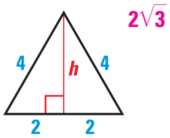
✓ GUIDED PRACTICE for Examples 4, 5, and 6

Find the value of the variable.

5. **3**



6. **$2\sqrt{3}$**



7. **WHAT IF?** In Example 6, what is the height of the body of the dump truck if it is raised 30° above the frame? **7 ft**

8. In a 30°-60°-90° triangle, *describe* the location of the shorter side. *Describe* the location of the longer side? ***Sample answer:* The shorter side is adjacent to the 60° angle; the longer side is adjacent to the 30° angle.**

7.4 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 9, and 27
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 19, 22, 29, and 34

SKILL PRACTICE

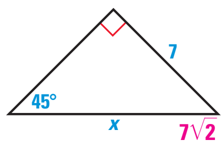
A

1. **VOCABULARY** Copy and complete: A triangle with two congruent sides and a right angle is called __?__. **an isosceles right triangle**

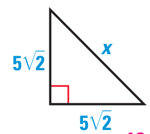
2. ★ **WRITING** *Explain* why the acute angles in an isosceles right triangle always measure 45°. **See margin.**

EXAMPLES 1 and 2 for Exs. 3–5

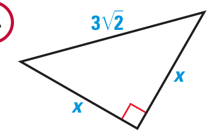
45°-45°-90° TRIANGLES **Find the value of x. Write your answer in simplest radical form.**

3.


$7\sqrt{2}$

4.


10

5.


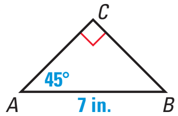
3

EXAMPLE 3 for Exs. 6–7

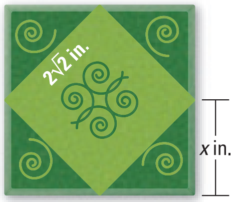
6. ★ **MULTIPLE CHOICE** Find the length of $\overline{AC}$. **C**

Ⓐ $7\sqrt{2}$ in. Ⓑ $2\sqrt{7}$ in.

Ⓒ $\frac{7\sqrt{2}}{2}$ in. Ⓓ $\sqrt{14}$ in.

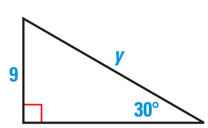


7. **ISOSCELES RIGHT TRIANGLE** The square tile shown has painted corners in the shape of congruent 45°-45°-90° triangles. What is the value of x? What is the side length of the tile? **2; 4 in.**

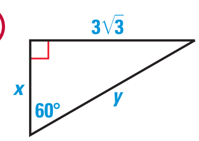


EXAMPLES 4 and 5 for Exs. 8–10

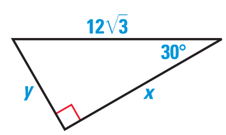
30°-60°-90° TRIANGLES **Find the value of each variable. Write your answers in simplest radical form.**

8.


$x = 9\sqrt{3}$, $y = 18$

9.


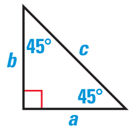
$x = 3$, $y = 6$

10.


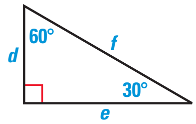
$x = 18$, $y = 6\sqrt{3}$

SPECIAL RIGHT TRIANGLES **Copy and complete the table.** **11, 12. See margin.**

B

11.


a	7	?	?	?	$\sqrt{5}$
b	?	11	?	?	?
c	?	?	10	$6\sqrt{2}$	?

12.


d	5	?	?	?	?
e	?	?	$8\sqrt{3}$	?	12
f	?	14	?	$18\sqrt{3}$	?

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1: SRH p. SR6 Exs. 17, 18, 22–24
Exs. 1–7, 11, 29, 30
Day 2:
Exs. 8–10, 12–19, 27, 28, 31

Average:
Day 1:
Exs. 1–7, 11, 29, 30, 33
Day 2:
Exs. 8–10, 12, 16–25, 27, 28, 31, 32

Advanced:
Day 1:
Exs. 1–7, 11, 29, 30, 33
Day 2:
Exs. 9, 10, 17–28*, 31, 32, 34, 35*

Block:
Exs. 1–7, 11, 29, 30, 33
(with previous lesson)
Exs. 8–10, 12, 16–25, 27, 28, 31, 32
(with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 8, 27, 28
Average: 4, 6, 9, 27, 29
Advanced: 4, 7, 10, 27, 30

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

2. The sum of the interior angles of a triangle is 180°; if one angle is 90°, then the other two angles must total 90°. Since the triangle is isosceles, these angles must be congruent. Therefore each angle must be half of 90° or 45°.

11.

a	7	11	$5\sqrt{2}$	6	$\sqrt{5}$
b	7	11	$5\sqrt{2}$	6	$\sqrt{5}$
c	$7\sqrt{2}$	$11\sqrt{2}$	10	$6\sqrt{2}$	$\sqrt{10}$

12.

d	5	7	8	$9\sqrt{3}$	$4\sqrt{3}$
e	$5\sqrt{3}$	$7\sqrt{3}$	$8\sqrt{3}$	27	12
f	10	14	16	$18\sqrt{3}$	$8\sqrt{3}$

Reading Strategy

Exercises 4–5 Even though no angle measures are given, students should notice that two sides of each triangle are congruent so they are 45°-45°-90° triangles.

Avoiding Common Errors

Exercise 10 Some students may be confused because they think that the $\sqrt{3}$ must be on the side of the triangle across from the 60° angle. Show students a triangle where the shorter leg is $2\sqrt{3}$, and have them see that the longer leg is 6 and the hypotenuse is $4\sqrt{3}$. Similarly, show them a 45°-45°-90° triangle where the length of a leg is a multiple of $\sqrt{2}$.

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Exercises 13–18**. This activity is also part of **Power Presentations**.

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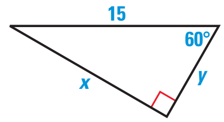
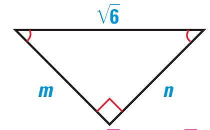
xy ALGEBRA **Find the value of each variable. Write your answers in simplest radical form.**

13.



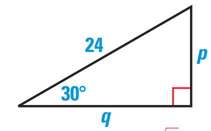
$x = \frac{15}{2}\sqrt{3}$, $y = \frac{15}{2}$

14.



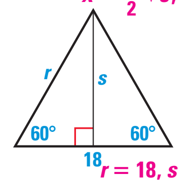
$m = \sqrt{3}$, $n = \sqrt{3}$

15.



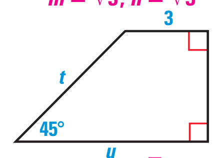
$p = 12$, $q = 12\sqrt{3}$

16.



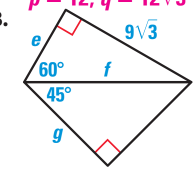
$r = 18$, $s = 9\sqrt{3}$

17.



$t = 4\sqrt{2}$, $u = 7$

18.



$e = 9$, $f = 18$, $g = 9\sqrt{2}$

Animated Geometry at my.hrw.com

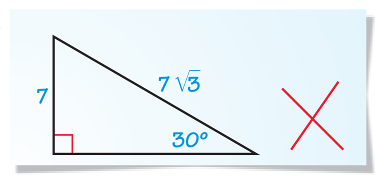
19. ★ **MULTIPLE CHOICE** Which side lengths do *not* represent a 30°-60°-90° triangle? **C**

Ⓐ $\frac{1}{2}, \frac{\sqrt{3}}{2}, 1$ Ⓑ $\sqrt{2}, \sqrt{6}, 2\sqrt{2}$

Ⓒ $\frac{5}{2}, \frac{5\sqrt{3}}{2}, 10$ Ⓓ $3, 3\sqrt{3}, 6$

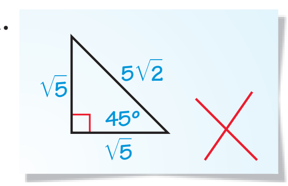
ERROR ANALYSIS ***Describe*** **and correct the error in finding the length of the hypotenuse.**

20.



20. The hypotenuse of a 30°-60°-90° triangle should be $2x$ not $x\sqrt{3}$; if $x = 7$, then the hypotenuse is 14.

21.

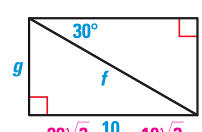


The hypotenuse of a 45°-45°-90° triangle should be $x\sqrt{2}$; if $x = \sqrt{5}$, then the hypotenuse is $\sqrt{10}$.

22. ★ **WRITING** Abigail solved Example 5 in a different way. Instead of dividing each side by $\sqrt{3}$, she multiplied each side by $\sqrt{3}$. Does her method work? *Explain* why or why not. **Yes.** ***Sample answer:*** **After she multiplies by $\sqrt{3}$ she would have to divide by 3 to solve for x and find $x = 3\sqrt{3}$.**

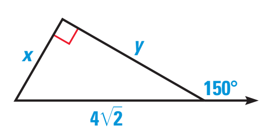
xy ALGEBRA **Find the value of each variable. Write your answers in simplest radical form.**

23.



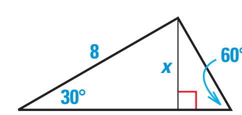
$f = \frac{20\sqrt{3}}{3}$, $g = \frac{10\sqrt{3}}{3}$

24.



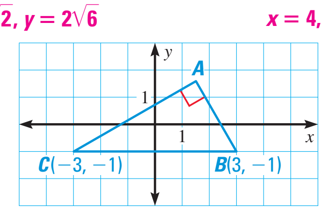
$x = 2\sqrt{2}$, $y = 2\sqrt{6}$

25.



$x = 4$, $y = \frac{4\sqrt{3}}{3}$

C **26.** **CHALLENGE** $\triangle ABC$ is a 30°-60°-90° triangle. Find the coordinates of A. **about (1.5, 1.60)**



○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

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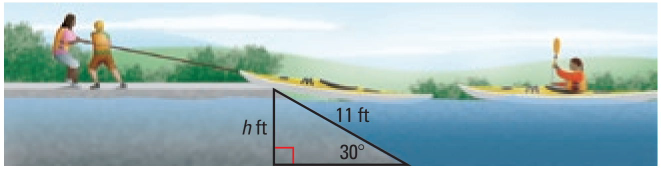
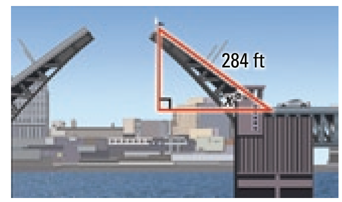
PROBLEM SOLVING

EXAMPLE 6 A for Ex. 27

27. KAYAK RAMP A ramp is used to launch a kayak. What is the height of an 11 foot ramp when its angle is 30° as shown? **5.5 ft**



28. DRAWBRIDGE Each half of the drawbridge is about 284 feet long, as shown. How high does a seagull who is on the end of the drawbridge rise when the angle with measure $x°$ is 30°? 45°? 60°?



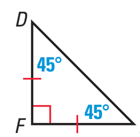
142 ft, $142\sqrt{2}$ ft, $142\sqrt{3}$ ft

29. ★ SHORT RESPONSE *Describe* two ways to show that all isosceles right triangles are similar to each other. **See margin.**

30. PROVING THEOREM 7.8 Write a paragraph proof of the 45°-45°-90° Triangle Theorem. **See margin.**

GIVEN ▶ $\triangle DEF$ is a 45°-45°-90° triangle.

PROVE ▶ The hypotenuse is $\sqrt{2}$ times as long as each leg.

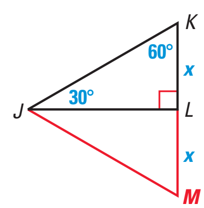


B **31. EQUILATERAL TRIANGLE** If an equilateral triangle has a side length of 20 inches, find the height of the triangle. **$10\sqrt{3}$ in.**

32. PROVING THEOREM 7.9 Write a paragraph proof of the 30°-60°-90° Triangle Theorem. **See margin.**

GIVEN ▶ $\triangle JKL$ is a 30°-60°-90° triangle.

PROVE ▶ The hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



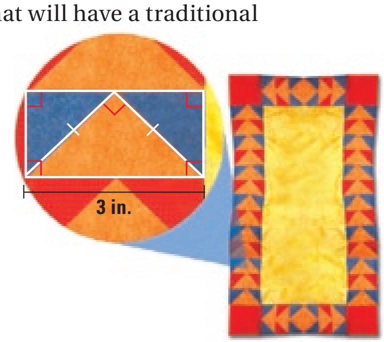
Plan for Proof Construct $\triangle JML$ congruent to $\triangle JKL$. Then prove that $\triangle JKM$ is equilateral. Express the lengths of $\overline{JK}$ and $\overline{JL}$ in terms of x.

33. MULTI-STEP PROBLEM You are creating a quilt that will have a traditional "flying geese" border, as shown below.

a. Find all the angle measures of the small blue triangles and the large orange triangles. **45°-45°-90° for all triangles**

b. The width of the border is to be 3 inches. To create the large triangle, you cut a square of fabric in half. Not counting any extra fabric needed for seams, what size square do you need? **$\frac{3\sqrt{2}}{2}$ in. × $\frac{3\sqrt{2}}{2}$ in.**

c. What size square do you need to create each small triangle? **1.5 in. × 1.5 in.**



Avoiding Common Errors

Exercise 32 Some students may write the square of the hypotenuse as $2x^2$, forgetting to write it as $(2x)^2$. Encourage them to start by writing $(\ \ )^2 + (\ \ )^2 = (\ \ )^2$, and then filling in expressions for the lengths of the three sides.

30. It is given that $\angle D \cong \angle E$, and $\angle F$ is a right angle, so by the Converse of the Base Angles Theorem, $\overline{DF} \cong \overline{EF}$. Then by the Pythagorean Theorem, $DF^2 + EF^2 = DE^2$. By substitution, the equation becomes $DF^2 + DF^2 = DE^2$. By addition, we get $2DF^2 = DE^2$ and a property of square roots allows us to state that $DE = DF \cdot \sqrt{2}$ or by substitution, $DE = EF \cdot \sqrt{2}$.

32. It is given that $\triangle JKL$ is a 30°-60°-90° triangle with x as the side opposite the 30° angle and $\triangle JKL \cong \triangle JML$. Since $m\angle KJL$ is 30° and $m\angle MJL$ is also 30°, angle addition shows that $\angle KJM$ measures 60°. In addition, the definition of an equiangular triangle shows that $\triangle JKM$ is equiangular and since $\triangle JKM$ is equiangular, it is also equilateral. This allows us to state that since $KM = 2x$, then $JK = 2x$. Therefore, JK which is the hypotenuse of $\triangle JKL$ is twice as long as the shorter leg of this triangle. Using $\triangle JKL$, the Pythagorean Theorem states that $JL^2 + LK^2 = JK^2$. The Substitution Property of Equality allows us to rewrite this equation as $JL^2 + x^2 = (2x)^2$. A property of exponents simplifies the equation to $JL^2 + x^2 = 4x^2$, and subtraction simplifies the equation to $JL^2 = 3x^2$. Finally, a property of square roots simplifies the equation to $JL = x\sqrt{3}$.

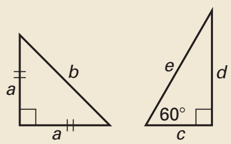
29. *Sample answer:* Method 1. Use the Angle-Angle Similarity Postulate, because by definition of an isosceles triangle, the base angles must be the same and in a right isosceles triangle, the angles are 45°. Method 2. Use the Side-Angle-Side Similarity Theorem, because a right angle is always congruent to another right angle and the ratio of the lengths of the corresponding sides of two isosceles right triangles will always be the same.

5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

Use these triangles for Exercises 1–4.



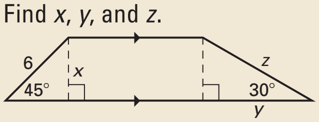
1. Find a if $b = 10\sqrt{2}$. **10**

2. Find b if $a = 19$. **$19\sqrt{2}$**

3. Find d and e if $c = 4$.
$d = 4\sqrt{3}$, $e = 8$

4. Find c and d if $e = 50\sqrt{3}$.
$c = 25\sqrt{3}$, $d = 75$

5. Find x, y, and z.



$x = 3\sqrt{2}$, $y = 3\sqrt{6}$, $z = 6\sqrt{2}$

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

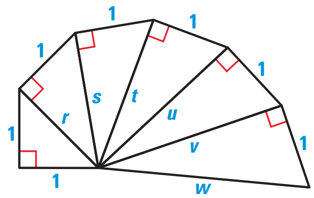
Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

34a, 35a. See Additional Answers.

34b. The left most triangle with sides of 1; the triangle must be a 45°–45°–90° triangle because it is an isosceles right triangle.

C **34.** ★ **EXTENDED RESPONSE** Use the figure at the right. You can use the fact that the converses of the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem are true.



a. Find the values of r, s, t, u, v, and w. *Explain* the procedure you used to find the values. **See margin.**

b. Which of the triangles, if any, is a 45°-45°-90° triangle? *Explain.*

c. Which of the triangles, if any, is a 30°-60°-90° triangle? *Explain.*
The triangle with t as the hypotenuse; the side lengths fit those given in the 30°-60°-90° Triangle Theorem

35. **CHALLENGE** In quadrilateral $QRST$, $m\angle R = 60°$, $m\angle T = 90°$, $QR = RS$, $ST = 8$, $TQ = 8$, and $\overline{RT}$ and $\overline{QS}$ intersect at point Z.

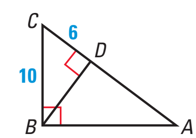
a. Draw a diagram. **See margin.**

b. *Explain* why $\triangle RQT \cong \triangle RST$. **Side-Side-Side Congruence Postulate**

c. Which is longer, QS or RT? *Explain.* **RT; $QS = 8\sqrt{2}$ and $RT = 4\sqrt{6} + 4\sqrt{2}$.**

QUIZ

In Exercises 1 and 2, use the diagram.

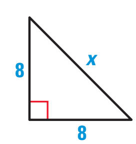


1. Which segment's length is the geometric mean of AC and CD? **CB**

2. Find BD, AD, and AB. **$8, \frac{32}{3}, \frac{40}{3}$**

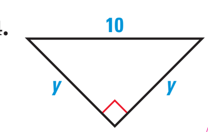
Find the values of the variable(s). Write your answer(s) in simplest radical form.

3.



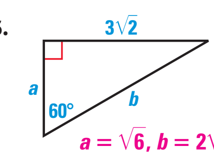
$8\sqrt{2}$

4.



$5\sqrt{2}$

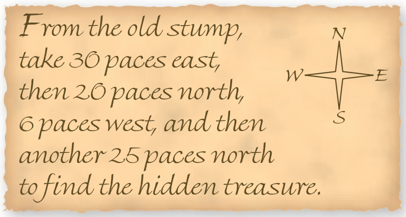
5.



$a = \sqrt{6}$, $b = 2\sqrt{6}$

MIXED REVIEW *of Problem Solving*

1. **GRIDDED ANSWER** Find the direct distance, in paces, from the treasure to the stump. **51**



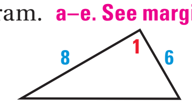
2. **MULTI-STEP PROBLEM** On a map of the United States, you put a pushpin on three state capitols you want to visit: Jefferson City, Missouri; Little Rock, Arkansas; and Atlanta, Georgia.

   a. Draw a diagram to model the triangle. **See margin.**

   b. Do the pushpins form a right triangle? If not, what type of triangle do they form? **no; obtuse triangle**

3. **SHORT RESPONSE** Bob and John started running at 10 A.M. Bob ran east at 4 miles per hour while John ran south at 5 miles per hour. How far apart were they at 11:30 A.M.? *Describe* how you calculated the answer. **See margin.**

4. **EXTENDED RESPONSE** Give all values of x that make the statement true for the given diagram. **a–e. See margin.**



   a. $\angle 1$ is a right angle. *Explain.*

   b. $\angle 1$ is an obtuse angle. *Explain.*

   c. $\angle 1$ is an acute angle. *Explain.*

   d. The triangle is isosceles. *Explain.*

   e. No triangle is possible. *Explain.*

5. **EXTENDED RESPONSE** A Chinese checker board is made of triangles. Use the picture below to answer the questions.

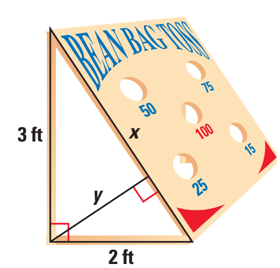
   a. Count the marble holes in the purple triangle. What kind of triangle is it? **equilateral triangle**

   b. If a side of the purple triangle measures 8 centimeters, find the area of the purple triangle. **$16\sqrt{3}$ cm^2**

   c. How many marble holes are in the center hexagon? Assuming each marble hole takes up the same amount of space, what is the relationship between the purple triangle and center hexagon? **See margin.**

   d. Find the area of the center hexagon. *Explain* your reasoning. **See margin.**

6. **MULTI-STEP PROBLEM** You build a beanbag toss game. The game is constructed from a sheet of plywood supported by two boards. The two boards form a right angle and their lengths are 3 feet and 2 feet.

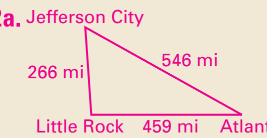


   a. Find the length x of the plywood. **$\sqrt{13}$ ft**

   b. You put in a support that is the altitude y to the hypotenuse of the right triangle. What is the length of the support? **$\frac{6\sqrt{13}}{13}$ ft**

   c. Where does the support attach to the plywood? *Explain.* **See margin.**

2a.



3. About 9.6 mi; I determined that they had been running 1.5 hours and used this to find the number of miles each person ran. These values form a right triangle. I then used the Pythagorean Theorem to find their distance apart.

4a. 10; if $m\angle 1 = 90°$, the Pythagorean Theorem gives 10.

4b. $10 < x < 14$; according to Theorem 4 the hypotenuse squared must be greater than the sum of the squares of the legs and the Triangle Inequality Theorem indicates that the side cannot exceed 14.

4c. $2 < x < 10$; according to Theorem3 the hypotenuse squared must be less than the sum of the squares of the legs and the Triangle Inequality Theorem indicates that the side must not be less than 2.

4d. 6 or 8; an isosceles triangle must have two sides of the same measure.

4e. $x < 2$ or $x > 14$; the Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle must be greater than the third.

5c. 61 marble holes; the purple triangle is $\frac{1}{6}$ of the center hexagon.

5d. $96\sqrt{3}$ cm^2; since the purple triangle is $\frac{1}{6}$ of the center hexagon and the purple triangle has an area of $16\sqrt{3}$ square centimeters, the area of the hexagon is $6 \cdot 16\sqrt{3} = 96\sqrt{3}$ square centimeters.

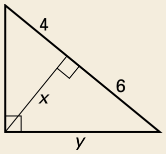
6c. $\frac{4\sqrt{13}}{13}$ feet from the bottom of the plywood along the hypotenuse; since the brace forms a right triangle with the front and base, the Pythagorean Theorem allows you to determine the distance from the ground.

1 PLAN AND PREPARE

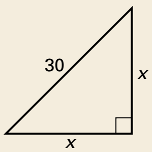
Warm-Up Exercises

Also available online

Find the values of the variables.

1. 

$x = 2\sqrt{6}$, $y = 2\sqrt{15}$

2. 

$15\sqrt{2}$

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day

Average: 1 day

Advanced: 1 day

Block: 0.5 block with previous lesson

• See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

How can you find a leg of a right triangle when you know the other leg and one acute angle? **Tell students they will learn how to answer this question by using the tangent ratio.**

7.5 Apply the Tangent Ratio

Before You used congruent or similar triangles for indirect measurement.

Now You will use the tangent ratio for indirect measurement.

Why? So you can find the height of a roller coaster, as in Ex. 32.

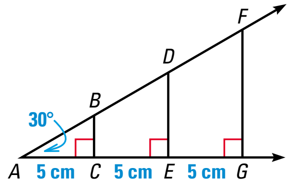
Key Vocabulary
- **trigonometric ratio**
- **tangent**

Step 3. Since $\triangle ABC \sim \triangle ADE$ by the AA Similarity Postulate, the proportions are true.

Step 4. The ratio of the lengths of the legs in similar right triangles is constant.

ACTIVITY RIGHT TRIANGLE RATIO

Materials: metric ruler, protractor, calculator



STEP 1 **Draw** a 30° angle and mark a point every 5 centimeters on a side as shown. Draw perpendicular segments through the 3 points.

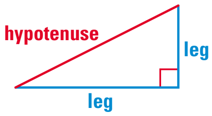
STEP 2 **Measure** the legs of each right triangle. Copy and complete the table.

Triangle	Adjacent leg	Opposite leg	$\frac{\text{Opposite leg}}{\text{Adjacent leg}}$
$\triangle ABC$	5 cm	about ? 2.9 cm	? 0.58
$\triangle ADE$	10 cm	about ? 5.8 cm	? 0.58
$\triangle AFG$	15 cm	about ? 8.7 cm	? 0.58

STEP 3 **Explain** why the proportions $\frac{BC}{DE} = \frac{AC}{AE}$ and $\frac{BC}{AC} = \frac{DE}{AE}$ are true.

STEP 4 **Make** a conjecture about the ratio of the lengths of the legs in a right triangle. Test your conjecture by using different acute angle measures.

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find the measure of a side or an acute angle in a right triangle.



The ratio of the lengths of the legs in a right triangle is constant for a given angle measure. This ratio is called the **tangent** of the angle.

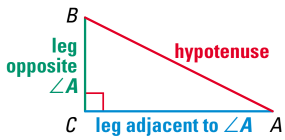
ABBREVIATE
Remember these abbreviations:
tangent → tan
opposite → opp.
adjacent → adj.

KEY CONCEPT *For Your Notebook*

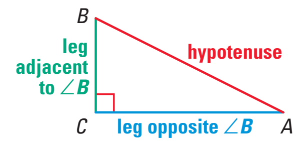
Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as tan A) is defined as follows:

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

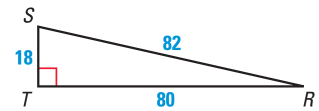


COMPLEMENTARY ANGLES In the right triangle, $\angle A$ and $\angle B$ are complementary so you can use the same diagram to find the tangent of $\angle A$ and the tangent of $\angle B$. Notice that the leg adjacent to $\angle A$ is the leg *opposite* $\angle B$ and the leg opposite $\angle A$ is the leg *adjacent* to $\angle B$.



EXAMPLE 1 Find tangent ratios

Find tan S and tan R. Write each answer as a fraction and as a decimal rounded to four places.



Solution

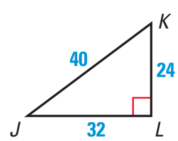
$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{80}{18} = \frac{40}{9} \approx 4.4444$$

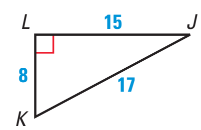
$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{18}{80} = \frac{9}{40} = 0.2250$$

APPROXIMATE
Unless told otherwise, you should round the values of trigonometric ratios to the ten-thousandths' place and round lengths to the tenths' place.

✓ GUIDED PRACTICE for Example 1

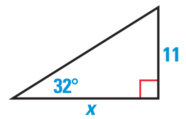
Find tan J and tan K. Round to four decimal places.

1.  0.7500, 1.3333

2.  0.5333, 1.8750

EXAMPLE 2 Find a leg length

ALGEBRA Find the value of x.



Solution

Use the tangent of an acute angle to find a leg length.

$\tan 32° = \frac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 32°.
$\tan 32° = \frac{11}{x}$	Substitute.
$x \cdot \tan 32° = 11$	Multiply each side by x.
$x = \frac{11}{\tan 32°}$	Divide each side by tan 32°.
$x \approx \frac{11}{0.6249}$	Use a calculator to find tan 32°.
$x \approx 17.6$	Simplify.

ANOTHER WAY
You can also use the Table of Trigonometric Ratios p. T7 to find the decimal values of trigonometric ratios.

Motivating the Lesson

A wire supports a tree. The wire is staked into the ground 10 feet from the tree and it forms an angle of 70° with the tree. In this lesson students will learn how to use the tangent ratio to determine how high up the tree the wire is attached.

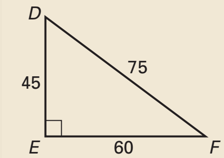
3 TEACH

Activity Note

It may be helpful to some students to identify each opposite leg by name $(\overline{BC}, \overline{DE}, \overline{FG})$ before entering the measurements in their tables. Also, point out that the 60° angles formed at points B, D, and F, can be used to complete the second part of Step 4.

Extra Example 1

Find tan D and tan F. Write each answer as a fraction and as a decimal rounded to four places.



$\tan D = \frac{4}{3} \approx 1.3333;$

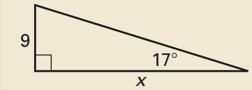
$\tan F = \frac{3}{4} = 0.7500$

Key Questions to Ask for Example 1

- Describe side $\overline{ST}$ in two ways. **It is the leg adjacent to $\angle S$ and it is the leg opposite $\angle R$.**
- Describe side $\overline{TR}$ in two ways. **It is the leg adjacent to $\angle R$ and it is the leg opposite $\angle S$.**

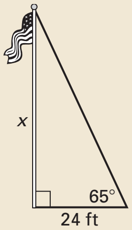
Extra Example 2

Find the value of x. **29.4**



Extra Example 3

Find the height of the flagpole to the nearest foot. **51 ft**



Extra Example 4

Use a special right triangle to find the tangent of a 45° angle. **tan 45° = 1**

Key Question to Ask for Example 4

- What is the tangent of a 30° angle?

$\tan 30° = \frac{\sqrt{3}}{3}$

Closing the Lesson

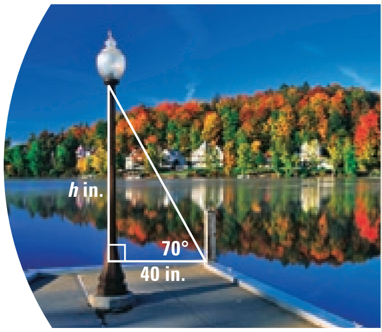
Have students summarize the major points of the lesson and answer the Essential Question: How can you find a leg of a right triangle when you know the other leg and one acute angle?

- **The tangent ratio for an acute angle of a right triangle is the length of the leg opposite the angle divided by the length of the leg adjacent to the angle.**

Use the tangent ratio. Solve for the unknown side length.

EXAMPLE 3 Estimate height using tangent

LAMPPOST Find the height h of the lamppost to the nearest inch.



$\tan 70° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tangent of 70°.

$\tan 70° = \frac{h}{40}$ Substitute.

$40 \cdot \tan 70° = h$ Multiply each side by 40.

$109.9 \approx h$ Use a calculator to simplify.

▶ The lamppost is about 110 inches tall.

SPECIAL RIGHT TRIANGLES You can find the tangent of an acute angle measuring 30°, 45°, or 60° by applying what you know about special right triangles.

EXAMPLE 4 Use a special right triangle to find a tangent

Use a special right triangle to find the tangent of a 60° angle.

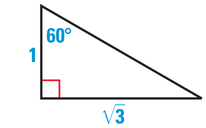
SIMILAR TRIANGLES
The tangents of all 60° angles are the same constant ratio. Any right triangle with a 60° angle can be used to determine this value.

STEP 1 **Because** all 30°-60°-90° triangles are similar, you can simplify your calculations by choosing 1 as the length of the shorter leg. Use the 30°-60°-90° Triangle Theorem to find the length of the longer leg.

longer leg = shorter leg $\cdot \sqrt{3}$ 30°-60°-90° Triangle Theorem

$x = 1 \cdot \sqrt{3}$ Substitute.

$x = \sqrt{3}$ Simplify.



STEP 2 **Find** tan 60°.

$\tan 60° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tangent of 60°.

$\tan 60° = \frac{\sqrt{3}}{1}$ Substitute.

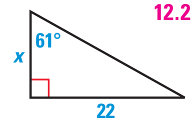
$\tan 60° = \sqrt{3}$ Simplify.

▶ The tangent of any 60° angle is $\sqrt{3} \approx 1.7321$.

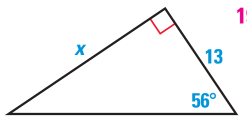
✓ GUIDED PRACTICE for Examples 2, 3, and 4

Find the value of x. Round to the nearest tenth.

3. **12.2**



4. **19.3**



5. WHAT IF? In Example 4, suppose the side length of the shorter leg is 5 instead of 1. Show that the tangent of 60° is still equal to $\sqrt{3}$.
shorter leg = 5, longer leg = $5\sqrt{3}$, $\tan 60° = \frac{5\sqrt{3}}{5} = \sqrt{3}$

7.5 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 7, and 31
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 17, 35, and 37

SKILL PRACTICE

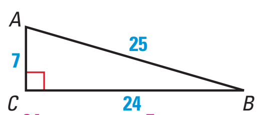
A

1. **VOCABULARY** Copy and complete: The tangent ratio compares the length of _?_ to the length of _?_. **the opposite leg, the adjacent leg**

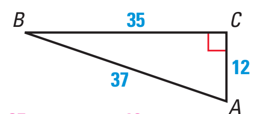
2. ★ **WRITING** *Explain* how you know that all right triangles with an acute angle measuring $n°$ are similar to each other. **See margin.**

EXAMPLE 1 for Exs. 3–5

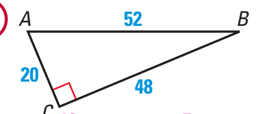
FINDING TANGENT RATIOS **Find tan A and tan B. Write each answer as a fraction and as a decimal rounded to four places.**

3. 

$\frac{24}{7}$ or 3.4286, $\frac{7}{24}$ or 0.2917

4. 

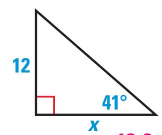
$\frac{35}{12}$ or 2.9167, $\frac{12}{35}$ or 0.3429

5. 

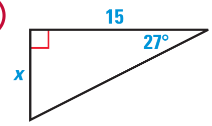
$\frac{12}{5}$ or 2.4000, $\frac{5}{12}$ or 0.4167

EXAMPLE 2 for Exs. 6–8

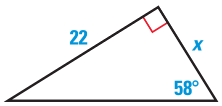
FINDING LEG LENGTHS **Find the value of x to the nearest tenth.**

6. 

13.8

7. 

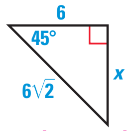
7.6

8. 

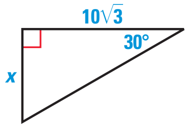
13.7

EXAMPLE 4 for Exs. 9–12

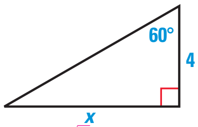
FINDING LEG LENGTHS **Find the value of x using the definition of tangent. Then find the value of x using the 45°-45°-90° Theorem or the 30°-60°-90° Theorem. *Compare* the results.**

9. 

6; 6; they are the same.

10. 

10; 10; they are the same.

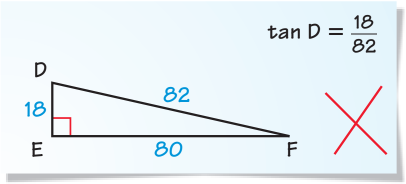
11. 

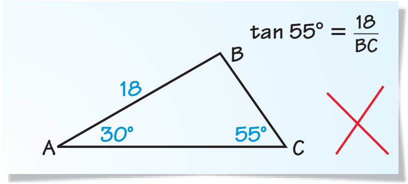
6.9282; $4\sqrt{3}$; they are the same.

12. **SPECIAL RIGHT TRIANGLES** Find tan 30° and tan 45° using the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem. $\tan 30 = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$, $\tan 45 = \frac{x}{x} = 1$

B

ERROR ANALYSIS ***Describe*** **the error in the statement of the tangent ratio. Correct the statement, if possible. Otherwise, write *not possible.***

13. 

14. 

13. Tangent is the ratio of the opposite and the adjacent side, not adjacent to hypotenuse; $\frac{80}{18}$.

14. The triangle is not a right triangle and the tangent ratio only applies to right triangles; not possible.

15. ★ **WRITING** *Describe* what you must know about a triangle in order to use the tangent ratio. **You need to know: that the triangle is a right triangle, which angle you will be applying the ratio to, and the lengths of the opposite side and the adjacent side to the angle.**

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1: SRH p. SR6 Exs. 25–28
Exs. 1–20, 31–35

Average:
Day 1:
Exs. 1, 2, 4–12 even, 13–26, 31–37

Advanced:
Day 1:
Exs. 1, 4–12 even, 15–30*, 32–38*

Block:
Exs. 1, 2, 4–12 even, 13–26, 31–37 (with previous lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 9, 31, 32
Average: 4, 6, 10, 32, 33
Advanced: 4, 8, 12, 32, 34

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

2. ***Sample answer:*** **All right triangles with an acute angle measuring $n°$ also have an acute angle measuring $(90 - n)°$. Therefore all the triangles with these measures will be similar.**

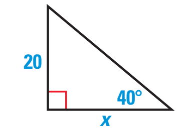
Avoiding Common Errors

Exercise 6 After the students write $\tan 41° = \frac{12}{x}$, they may mistakenly write "$x = \frac{\tan 41°}{12}$". Have them multiply both sides of $\tan 41° = \frac{12}{x}$ by x to get $x \cdot \tan 41° = 12$ and then divide by $\tan 41°$ to get $x = \frac{12}{\tan 41°}$.

Reading Strategy

Exercises 27–29 Encourage students to look carefully at the diagrams and to notice that the large triangles are not marked as right triangles.

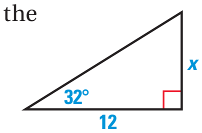
16. ★ **MULTIPLE CHOICE** Which expression can be used to find the value of x in the triangle shown? **C**



Ⓐ $x = 20 \cdot \tan 40°$ Ⓑ $x = \frac{\tan 40°}{20}$

Ⓒ $x = \frac{20}{\tan 40°}$ Ⓓ $x = \frac{20}{\tan 50°}$

17. ★ **MULTIPLE CHOICE** What is the approximate value of x in the triangle shown? **C**

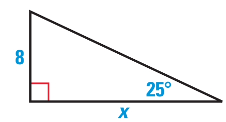


Ⓐ 0.4 Ⓑ 2.7

Ⓒ 7.5 Ⓓ 19.2

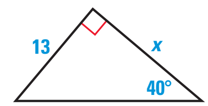
FINDING LEG LENGTHS **Use a tangent ratio to find the value of x. Round to the nearest tenth. Check your solution using the tangent of the other acute angle.**

18.



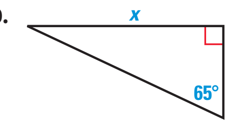
17.2

19.



15.5

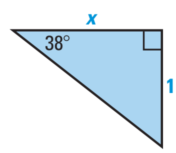
20.



19.3

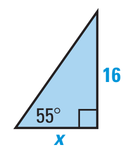
FINDING AREA **Find the area of the triangle. Round to the nearest tenth.**

21.



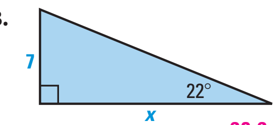
77.4

22.



89.6

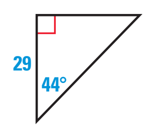
23.



60.6

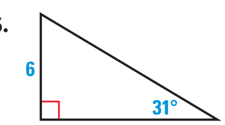
FINDING PERIMETER **Find the perimeter of the triangle. Round to the nearest tenth.**

24.



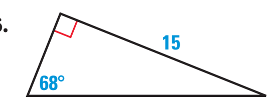
97.3

25.



27.6

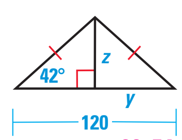
26.



37.2

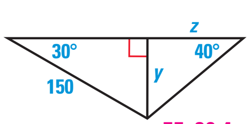
C **FINDING LENGTHS** **Find y. Then find z. Round to the nearest tenth.**

27.



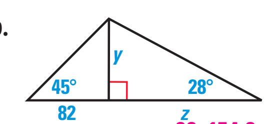
60; 54.0

28.



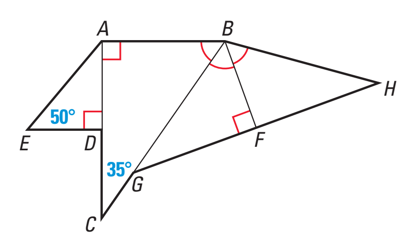
75; 89.4

29.



82; 154.2

30. **CHALLENGE** Find the perimeter of the figure at the right, where $AC = 26$, $AD = BF$, and D is the midpoint of $\overline{AC}$. about 128



◯ = See **WORKED-OUT SOLUTIONS** in Student Resources

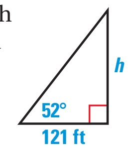
★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 3 A
for Exs. 31–32

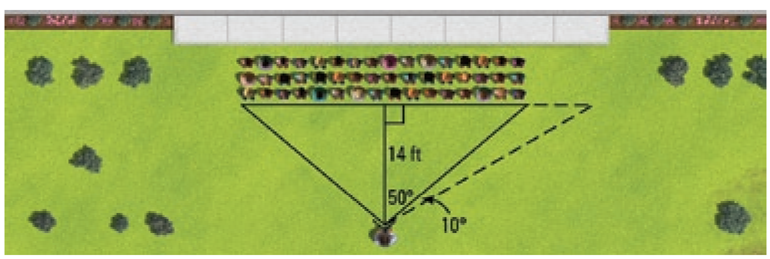
31. **WASHINGTON MONUMENT** A surveyor is standing 118 feet from the base of the Washington Monument. The surveyor measures the angle between the ground and the top of the monument to be 78°. Find the height h of the Washington Monument to the nearest foot. **555 ft**

32. **ROLLER COASTERS** A roller coaster makes an angle of 52° with the ground. The horizontal distance from the crest of the hill to the bottom of the hill is about 121 feet, as shown. Find the height h of the roller coaster to the nearest foot. **155 ft**



CLASS PICTURE Use this information and diagram for Exercises 33 and 34.

Your class is having a class picture taken on the lawn. The photographer is positioned 14 feet away from the center of the class. If she looks toward either end of the class, she turns 50°.



33. **ISOSCELES TRIANGLE** What is the distance between the ends of the class? **about 33.4 ft**

34. **MULTI-STEP PROBLEM** The photographer wants to estimate how many more students can fit at the end of the first row. The photographer turns 50° to see the last student and another 10° to see the end of the camera range.

   a. Find the distance from the center to the last student in the row. **about 16.7 ft**

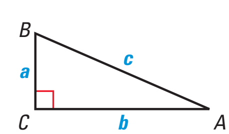
   b. Find the distance from the center to the end of the camera range. **about 24.2 ft**

   c. Use the results of parts (a) and (b) to estimate the length of the empty space. **about 7.5 ft**

   d. If each student needs 2 feet of space, about how many more students can fit at the end of the first row? *Explain* your reasoning. **3; 7.5 ÷ 2 is 3.75, but there is not enough room for 4 students, so round down.**

B

35. ★ **SHORT RESPONSE** Write expressions for the tangent of each acute angle in the triangle. *Explain* how the tangent of one acute angle is related to the tangent of the other acute angle. What kind of angle pair are $\angle A$ and $\angle B$?
$\tan A = \frac{a}{b}$, $\tan B = \frac{b}{a}$; the tangent of one acute angle is the reciprocal of the tangent of the other acute angle; complementary.



Study Strategy

Exercise 34 Have students draw and label a diagram for the problem. Remind them that in order to use the tangent ratio they must have a right triangle. Therefore, for part (c) finding the empty space means finding the difference of the values they found in part (a) and part (b).

Internet Reference

Exercise 31 More information about the Washington Monument can be found by visiting www.nps.gov/wamo/

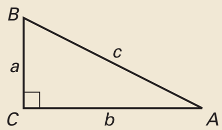
Exercise 37 Additional information about the Americans with Disabilities Act can be found at www.usdoj.gov/crt/ada/adahom1.htm

5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

Use this diagram for Exercises 1–4.



1. If $a = 18$, $b = 80$, and $c = 82$, find tan B and tan A. Write each answer as a fraction and a decimal rounded to 4 places.
$\tan B = \frac{40}{9} \approx 4.4444;$
$\tan A = \frac{9}{40} = 0.2250$

2. If $a = 17$ and $m\angle A = 31°$, find b to the nearest tenth. **28.3**

3. If $a = 9$ and $m\angle B = 74°$, find b to the nearest tenth. **31.4**

4. If $a = 5$ and $m\angle B = 79°$, find the number of square units in the area of $\triangle ABC$ to the nearest tenth. **64.3**

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

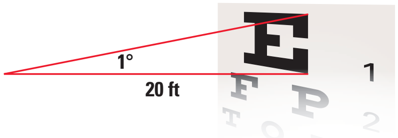
- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

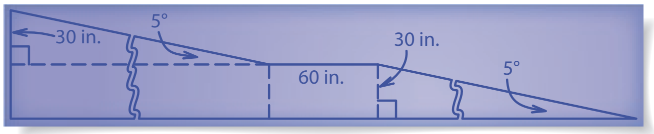
37b. See Additional Answers.

36. EYE CHART You are looking at an eye chart that is 20 feet away. Your eyes are level with the bottom of the "E" on the chart. To see the top of the "E," you look up 1°. How tall is the "E"? **about 4.2 in.**



Not drawn to scale

37. ★ EXTENDED RESPONSE According to the Americans with Disabilities Act, a ramp cannot have an incline that is greater than 5°. The regulations also state that the maximum rise of a ramp is 30 inches. When a ramp needs to reach a height greater than 30 inches, a series of ramps connected by 60 inch landings can be used, as shown below.

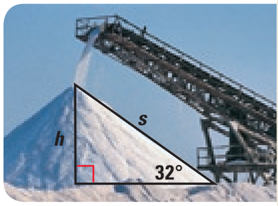


a. What is the maximum horizontal length of the base of one ramp, in feet? Round to the nearest foot. **29 ft**

b. If a doorway is 7.5 feet above the ground, what is the least number of ramps and landings you will need to lead to the doorway? Draw and label a diagram to *justify* your answer. **3 ramps and 2 landings; see margin for art.**

c. To the nearest foot, what is the total length of the base of the system of ramps and landings in part (b)? **96 ft**

C **38. CHALLENGE** The road salt shown is stored in a cone-shaped pile. The base of the cone has a circumference of 80 feet. The cone rises at an angle of 32°. Find the height h of the cone. Then find the length s of the cone-shaped pile. **about 8 ft; about 15 ft**



7.6 Apply the Sine and Cosine Ratios

Before You used the tangent ratio.
Now You will use the sine and cosine ratios.
Why So you can find distances, as in Ex. 39.

Key Vocabulary
- **sine**
- **cosine**
- **angle of elevation**
- **angle of depression**

ABBREVIATE
Remember these abbreviations:
sine → sin
cosine → cos
hypotenuse → hyp

The **sine** and **cosine** ratios are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

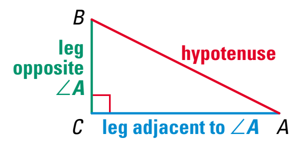
KEY CONCEPT *For Your Notebook*

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written $\sin A$ and $\cos A$) are defined as follows:

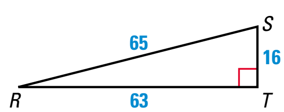
$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$



EXAMPLE 1 Find sine ratios

Find sin *S* and sin *R*. Write each answer as a fraction and as a decimal rounded to four places.



Solution

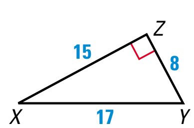
$$\sin S = \frac{\text{opp. } \angle S}{\text{hyp.}} = \frac{RT}{SR} = \frac{63}{65} \approx 0.9692$$

$$\sin R = \frac{\text{opp. } \angle R}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{65} \approx 0.2462$$

✓ **GUIDED PRACTICE** for Example 1

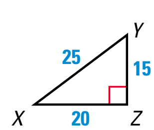
Find sin *X* and sin *Y*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

1.



$\frac{8}{17}$ or 0.4706, $\frac{15}{17}$ or 0.8824

2.



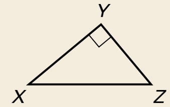
$\frac{3}{5}$ or 0.6, $\frac{4}{5}$ or 0.8

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Use this diagram for Exercises 1–4.



1. Name the hypotenuse. $\overline{XZ}$
2. Name the leg opposite $\angle X$. $\overline{YZ}$
3. Name the leg adjacent to $\angle X$. $\overline{XY}$
4. If $XY = 17$ and $m\angle X = 41°$, find YZ. 14.78

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
- See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

How can you find the lengths of the sides of a right triangle when you are given the length of the hypotenuse and one acute angle? **Tell students they will learn how to answer this question by using trigonometric ratios.**

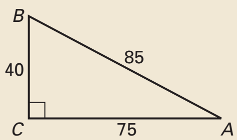
Motivating the Lesson

A long ladder rests against a building, forming an angle of 80° at the ground. If you know one of three lengths—the ladder, the horizontal distance from the base of the building to the ladder, or the height reached by the ladder—you can use the methods in this lesson to find the other two lengths.

3 TEACH

Extra Example 1

Find sin *A* and sin *B*. Write each answer as a fraction and as a decimal rounded to four places.

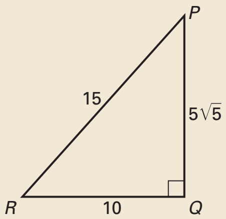


$\sin A = \frac{8}{17} \approx 0.4706$;

$\sin B = \frac{15}{17} \approx 0.8824$

Extra Example 2

Find cos *P* and cos *R*. Write each answer as a fraction and as a decimal.

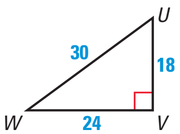


$\cos P = \frac{\sqrt{5}}{3} \approx 0.7454$;

$\cos R = \frac{2}{3} \approx 0.6667$

EXAMPLE 2 Find cosine ratios

Find cos *U* and cos *W*. Write each answer as a fraction and as a decimal.



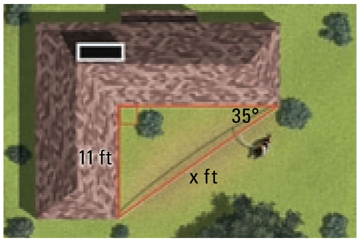
Solution

$$\cos U = \frac{\text{adj. to } \angle U}{\text{hyp.}} = \frac{UV}{UW} = \frac{18}{30} = \frac{3}{5} = 0.6000$$

$$\cos W = \frac{\text{adj. to } \angle W}{\text{hyp.}} = \frac{WV}{UW} = \frac{24}{30} = \frac{4}{5} = 0.8000$$

EXAMPLE 3 Use a trigonometric ratio to find a hypotenuse

DOG RUN You want to string cable to make a dog run from two corners of a building, as shown in the diagram. Write and solve a proportion using a trigonometric ratio to approximate the length of cable you will need.



Solution

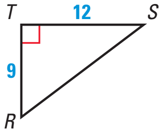
$\sin 35° = \frac{\text{opp.}}{\text{hyp.}}$	Write ratio for sine of 35°.
$\sin 35° = \frac{11}{x}$	Substitute.
$x \cdot \sin 35° = 11$	Multiply each side by *x*.
$x = \frac{11}{\sin 35°}$	Divide each side by sin 35°.
$x \approx \frac{11}{0.5736}$	Use a calculator to find sin 35°.
$x \approx 19.2$	Simplify.

▸ You will need a little more than 19 feet of cable.

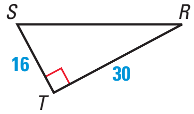
✓ GUIDED PRACTICE for Examples 2 and 3

In Exercises 3 and 4, find cos *R* and cos *S*. Write each answer as a decimal. Round to four decimal places, if necessary.

3. 0.6, 0.8



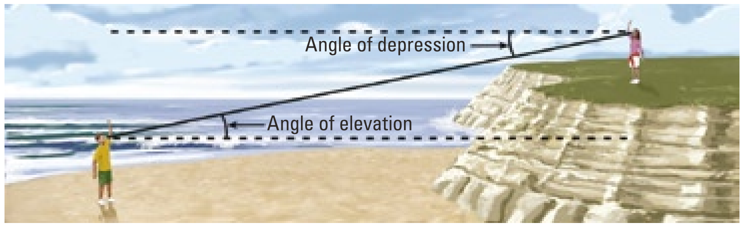
4. 0.8824, 0.4706



5. In Example 3, use the cosine ratio to find the length of the other leg of the triangle formed. about 15.7 ft

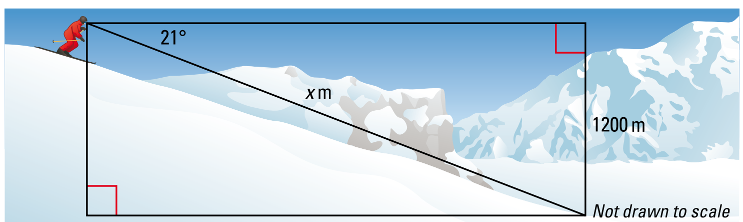
ANGLES If you look up at an object, the angle your line of sight makes with a horizontal line is called the **angle of elevation**. If you look down at an object, the angle your line of sight makes with a horizontal line is called the **angle of depression**.

APPLY THEOREMS
Notice that the angle of elevation and the angle of depression are congruent by the Alternate Interior Angles Theorem.



EXAMPLE 4 Find a hypotenuse using an angle of depression

SKIING You are skiing on a mountain with an altitude of 1200 meters. The angle of depression is 21°. About how far do you ski down the mountain?



Solution

$\sin 21° = \dfrac{\text{opp.}}{\text{hyp.}}$ **Write ratio for sine of 21°.**

$\sin 21° = \dfrac{1200}{x}$ **Substitute.**

$x \cdot \sin 21° = 1200$ **Multiply each side by *x*.**

$x = \dfrac{1200}{\sin 21°}$ **Divide each side by sin 21°.**

$x \approx \dfrac{1200}{0.3584}$ **Use a calculator to find sin 21°.**

$x \approx 3348.2$ **Simplify.**

▶ You ski about 3348 meters down the mountain.

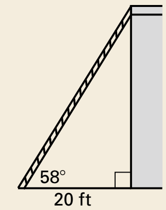
Animated Geometry at my.hrw.com

✓ **GUIDED PRACTICE** for Example 4

6. **WHAT IF?** Suppose the angle of depression in Example 4 is 28°. About how far would you ski? **about 2556 m**

Extra Example 3

A rope, staked 20 feet from the base of a building, goes to the roof and forms an angle of 58° with the ground. To the nearest tenth of a foot, how long is the rope? **37.7 ft**

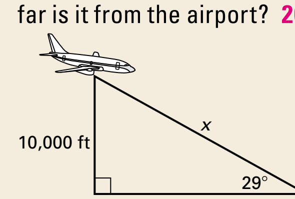


Key Question to Ask for Example 3

- How could you solve this problem using the cosine ratio? *Explain.* **The other acute angle is 55°, so $\cos 55° = \frac{11}{x}$. Then $x \cdot \cos 55° = 11$ and $x = \frac{11}{\cos 55°} \approx 19.2$ ft.**

Extra Example 4

A pilot is looking at an airport from her plane. The angle of depression is 29°. If the plane is at an altitude of 10,000 feet, approximately how far is it from the airport? **20,627 ft**



Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 4**. This activity is also part of **Power Presentations**.

Vocabulary

One memory device to remember the three trig ratios is "soh-cah-toa." This stands for **s**ine: **o**pposite over **h**ypotenuse; **c**osine: **a**djacent over **h**ypotenuse; **t**angent: **o**pposite over **a**djacent.

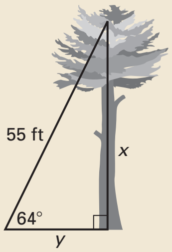
Differentiated Instruction

Advanced Have the students draw a circle with center (0, 0) and radius 1 (in trigonometry, this is called the "unit circle"). Have them draw a radius forming a 30° central angle with the positive *x*-axis. Identify the point (*x*, *y*) where the radius meets the circle. Ask them to find the values of *x* and *y*. Then have them repeat this procedure for a 45° central angle and a 60° central angle. The coordinates are $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 5

A dog is looking at a squirrel at the top of a tree. The distance between the two animals is 55 feet and the angle of elevation is 64°. How high is the squirrel and how far is the dog from the base of the tree?



49.4 ft high; 24.1 ft from tree

Extra Example 6

Use a special right triangle to find the sine and cosine of a 45° angle.

$\sin 45° = \frac{\sqrt{2}}{2}$; $\cos 45° = \frac{\sqrt{2}}{2}$

Closing the Lesson

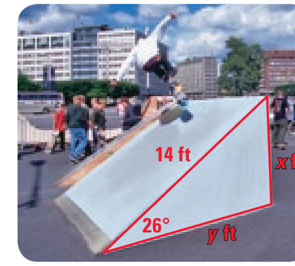
Have students summarize the major points of the lesson and answer the Essential Question: How can you find the lengths of the legs of a right triangle when you are given the length of the hypotenuse and one acute angle?

- **For acute angle A in a right triangle, $\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$.**
- **For acute angle A in a right triangle, $\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$.**

If you know one of the acute angles in a right triangle and the length of the hypotenuse, you can use the sine ratio and cosine ratio to find the lengths of the other two sides.

EXAMPLE 5 Find leg lengths using an angle of elevation

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and length of the base of the ramp.



ANOTHER WAY
For alternative methods for solving the problem in Example 5, see the **Problem Solving Workshop**.

Solution

STEP 1 **Find** the height.

$\sin 26° = \frac{\text{opp.}}{\text{hyp.}}$ Write ratio for sine of 26°.

$\sin 26° = \frac{x}{14}$ Substitute.

$14 \cdot \sin 26° = x$ Multiply each side by 14.

$6.1 \approx x$ Use a calculator to simplify.

▶ The height is about 6.1 feet.

STEP 2 **Find** the length of the base.

$\cos 26° = \frac{\text{adj.}}{\text{hyp.}}$ Write ratio for cosine of 26°.

$\cos 26° = \frac{y}{14}$ Substitute.

$14 \cdot \cos 26° = y$ Multiply each side by 14.

$12.6 \approx y$ Use a calculator to simplify.

▶ The length of the base is about 12.6 feet.

EXAMPLE 6 Use a special right triangle to find a sine and cosine

Use a special right triangle to find the sine and cosine of a 60° angle.

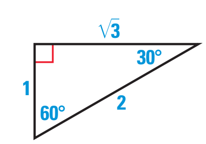
DRAW DIAGRAMS
To simplify calculations you can choose 1 as the length of the shorter leg.

Solution

Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and 2. Then set up sine and cosine ratios for the 60° angle.

$$\sin 60° = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$

$$\cos 60° = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{2} = 0.5000$$



✓ GUIDED PRACTICE for Examples 5 and 6

7. **WHAT IF?** In Example 5, suppose the angle of elevation is 35°. What is the new height and base length of the ramp? **about 8 ft, about 11.5 ft**

8. Use a special right triangle to find the sine and cosine of a 30° angle. **$\frac{1}{2}$, $\frac{\sqrt{3}}{2}$**

Paul A. Souders/Corbis

Differentiated Instruction

English Learners Distinguishing between *sine* and *cosine* may be challenging for some students. Explain that the prefix *co-* can mean "together" as it does in the word cooperate. Point out that the cosine ratio for an acute angle of a triangle involves the adjacent leg. Tell students to remember this by thinking of the adjacent leg as coming "together" with the hypotenuse to form the angle.

See also the *Differentiated Instruction Resources* for more strategies.

7.6 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 9, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 18, 29, 35, and 37

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

SKILL PRACTICE

A

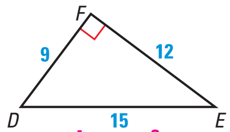
1. VOCABULARY Copy and complete: The sine ratio compares the length of __?__ to the length of __?__. **the opposite leg, the hypotenuse**

2. ★ WRITING *Explain* how to tell which side of a right triangle is adjacent to an angle and which side is the hypotenuse. **The adjacent side is the side that forms part of the angle and is not opposite the right angle; the hypotenuse is the side opposite the right angle.**

EXAMPLE 1 for Exs. 3–6

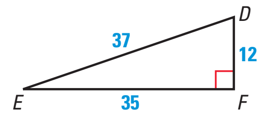
FINDING SINE RATIOS Find sin *D* and sin *E*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

3.



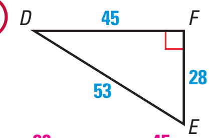
$\frac{4}{5}$ **or 0.8,** $\frac{3}{5}$ **or 0.6**

4.



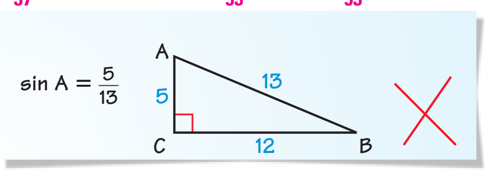
$\frac{35}{37}$ **or 0.9459,** $\frac{12}{37}$ **or 0.3243**

5.



$\frac{28}{53}$ **or 0.5283,** $\frac{45}{53}$ **or 0.8491**

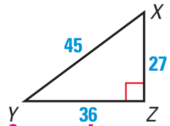
6. ERROR ANALYSIS *Explain* why the student's statement is incorrect. Write a correct statement for the sine of the angle. **The ratio for sine is opposite over hypotenuse, not adjacent over hypotenuse;** $\sin A = \frac{12}{13}$.



EXAMPLE 2 for Exs. 7–9

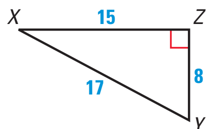
FINDING COSINE RATIOS Find cos *X* and cos *Y*. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.

7.



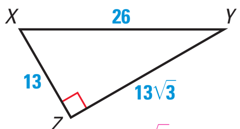
$\frac{3}{5}$ **or 0.6,** $\frac{4}{5}$ **or 0.8**

8.



$\frac{15}{17}$ **or 0.8824,** $\frac{8}{17}$ **or 0.4706**

9.

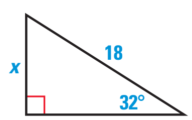


$\frac{1}{2}$ **or 0.5,** $\frac{\sqrt{3}}{2}$ **or 0.8660**

EXAMPLE 3 for Exs. 10–15

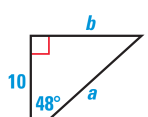
USING SINE AND COSINE RATIOS Use a sine or cosine ratio to find the value of each variable. Round decimals to the nearest tenth.

10.



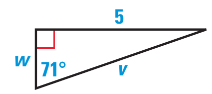
$x = 9.5, y = 15.3$

11.



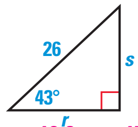
$a = 14.9, b = 11.1$

12.



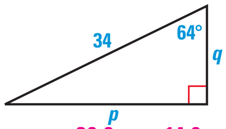
$v = 5.3, w = 1.7$

13.



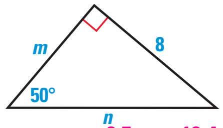
$r = 19.0, s = 17.7$

14.



$p = 30.6, q = 14.9$

15.



$m = 6.7, n = 10.4$

EXAMPLE 6 for Ex. 16

16. SPECIAL RIGHT TRIANGLES Use the 45°-45°-90° Triangle Theorem to find the sine and cosine of a 45° angle. $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–15
Day 2:
Exs. 16–21, 33–38

Average:
Day 1:
Exs. 1, 2, 4–6, 8, 9, 11–13, 19–27
Day 2:
Exs. 16–18, 28, 29, 34–39

Advanced:
Day 1:
Exs. 1, 2, 4, 5, 8, 9, 13–15, 19–27, 30, 31
Day 2:
Exs. 16–18, 28, 29, 32*, 35–41*

Block:
Exs. 1, 2, 4–6, 8, 9, 11–13, 16–29, 34–39

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 7, 10, 34, 35
Average: 4, 8, 12, 34, 36
Advanced: 4, 8, 14, 35, 36

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

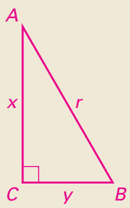
Graphing Calculator

Exercise 16 Have the students find the sine and cosine of a 30°, 45°, and 60° angle on their calculator. Have them compare these values to the ones they found using special right triangles.

Mathematical Reasoning

Exercise 29 Students should understand that the sine or cosine of an angle cannot be greater than 1 because it is the ratio of a leg length to the hypotenuse length, and the hypotenuse is the longest side in a right triangle.

32a.



$\sin A = \frac{y}{r} \rightarrow y = r \sin A$

$\cos A = \frac{x}{r} \rightarrow x = r \cos A$

$\tan A = \frac{y}{x} = \frac{\sin A}{\cos A}$

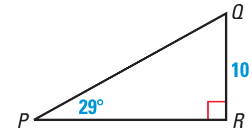
32b.

$$x^2 + y^2 = r^2$$
$$(r \cos A)^2 + (r \sin A)^2 = r^2$$
$$r^2 \cos^2 A + r^2 \sin^2 A = r^2$$
$$r^2(\cos^2 A + \sin^2 A) = r^2$$
$$\cos^2 A + \sin^2 A = 1$$

B

17. ★ **WRITING** *Describe* what you must know about a triangle in order to use the sine ratio and the cosine ratio. **The triangle must be a right triangle, and you need either an acute angle measure and the length of one side or the lengths of two sides of the triangle.**

18. ★ **MULTIPLE CHOICE** In $\triangle PQR$, which expression can be used to find PQ? **C**

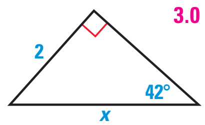


Ⓐ $10 \cdot \cos 29°$ Ⓑ $10 \cdot \sin 29°$

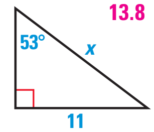
Ⓒ $\frac{10}{\sin 29°}$ Ⓓ $\frac{10}{\cos 29°}$

ALGEBRA **Find the value of x. Round decimals to the nearest tenth.**

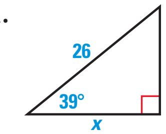
19. **3.0**



20. **13.8**

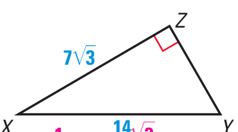


21. **20.2**



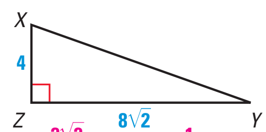
FINDING SINE AND COSINE RATIOS **Find the unknown side length. Then find sin X and cos X. Write each answer as a fraction in simplest form and as a decimal. Round to four decimal places, if necessary.**

22.



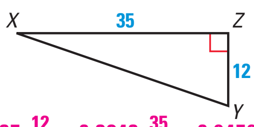
7; $\frac{1}{2}$ or 0.5, $\frac{\sqrt{3}}{2}$ or 0.8660

23.



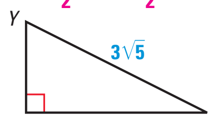
12; $\frac{2\sqrt{2}}{3}$ or 0.9428, $\frac{1}{3}$ or 0.3333

24.



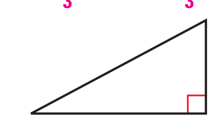
37; $\frac{12}{37}$ or 0.3243, $\frac{35}{37}$ or 0.9459

25.



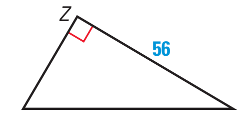
3; $\frac{\sqrt{5}}{5}$ or 0.4472, $\frac{2\sqrt{5}}{5}$ or 0.8944

26.



34; $\frac{8}{17}$ or 0.4706, $\frac{15}{17}$ or 0.8824

27.



33; $\frac{56}{65}$ or 0.8615, $\frac{33}{65}$ or 0.5077

28. **ANGLE MEASURE** Make a prediction about how you could use trigonometric ratios to find angle measures in a triangle. ***Sample answer:* You can use $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ to determine the angle measure when you have the appropriate ratio.**

29. ★ **MULTIPLE CHOICE** In $\triangle JKL$, $m\angle L = 90°$. Which statement about $\triangle JKL$ *cannot* be true? **D**

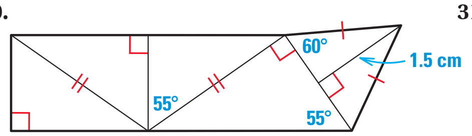
Ⓐ $\sin J = 0.5$ Ⓑ $\sin J = 0.1071$

Ⓒ $\sin J = 0.8660$ Ⓓ $\sin J = 1.1$

C

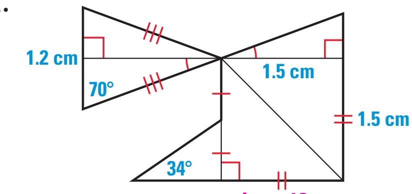
PERIMETER **Find the approximate perimeter of the figure.**

30.



about 14 cm

31.



about 13 cm

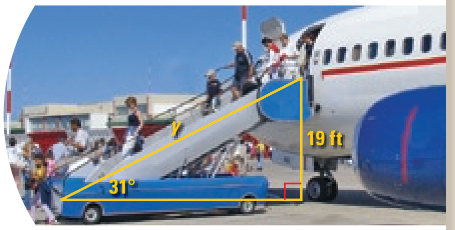
32. **CHALLENGE** Let A be any acute angle of a right triangle. Show that (a) $\tan A = \frac{\sin A}{\cos A}$ and (b) $(\sin A)^2 + (\cos A)^2 = 1$. **See margin.**

○ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

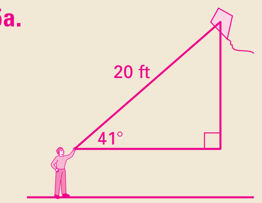
PROBLEM SOLVING

EXAMPLES 4 and 5 for Exs. 33–36 **A**

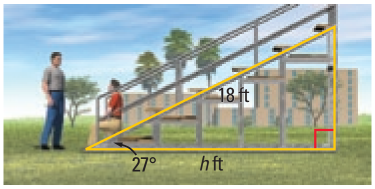
33. AIRPLANE RAMP The airplane door is 19 feet off the ground and the ramp has a 31° angle of elevation. What is the length y of the ramp? **about 36.9 ft**



35a.



34. BLEACHERS Find the horizontal distance h the bleachers cover. Round to the nearest foot. **16 ft**

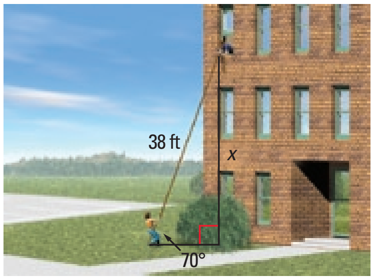


35. ★ SHORT RESPONSE You are flying a kite with 20 feet of string extended. The angle of elevation from the spool of string to the kite is 41°.

a. Draw and label a diagram to represent the situation. **See margin.**

b. How far off the ground is the kite if you hold the spool 5 feet off the ground? *Describe* how the height where you hold the spool affects the height of the kite. **About 18.1 ft; the height that the spool is off the ground has to be added.**

36. MULTI-STEP PROBLEM You want to hang a banner that is 29 feet tall from the third floor of your school. You need to know how tall the wall is, but there is a large bush in your way.

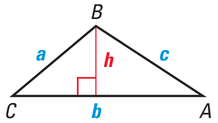


a. You throw a 38 foot rope out of the window to your friend. She extends it to the end and measures the angle of elevation to be 70°. How high is the window? **about 35.7 ft**

b. The bush is 6 feet tall. Will your banner fit above the bush? **yes**

c. What If? Suppose you need to find how far from the school your friend needs to stand. Which trigonometric ratio should you use? **cosine**

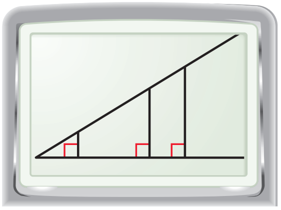
37. ★ SHORT RESPONSE Explain why the area of $\triangle ABC$ in the diagram can be found using the formula Area $= \frac{1}{2}ab \sin C$. Then calculate the area if $a = 4$, $b = 7$, and $m\angle C = 40°$.



37. $\sin C = \frac{h}{a}$, so $h = a \sin C$, and Area $= \frac{1}{2}bh = \frac{1}{2}b(a \sin C) = \frac{1}{2}ab \sin C$; about 9 square units.

B

38. TECHNOLOGY Use geometry drawing software to construct an angle. Mark three points on one side of the angle and construct segments perpendicular to that side at the points. Measure the legs of each triangle and calculate the sine of the angle. Is the sine the same for each triangle? **yes**



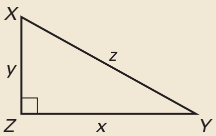
Profimedia.CZ s.r.o./Alamy

5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

Use this diagram for Exercises 1–3.



1. If $x = 4\sqrt{5}$, $y = 4$, and $z = 4\sqrt{6}$, find sin X, sin Y, cos X, and cos Y.

$\sin X = \frac{\sqrt{30}}{6} \approx 0.9129$,

$\sin Y = \frac{\sqrt{6}}{6} \approx 0.4082$,

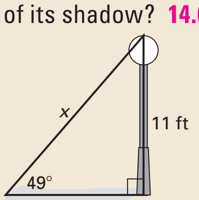
$\cos X = \frac{\sqrt{6}}{6} \approx 0.4082$,

$\cos Y = \frac{\sqrt{30}}{6} \approx 0.9129$

2. If $y = 10$ and $m\angle Y = 15°$, find z to the nearest tenth. **38.6**

3. If $z = 12$ and $m\angle X = 84°$, find y to the nearest tenth. **1.3**

4. A lamppost is 11 feet tall. If the angle of elevation through the top of the lamppost to the sun is 49°, approximately how far is the top of the lamppost from the tip of its shadow? **14.6 ft**



Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

39a–c, 40, 41a. See Additional Answers.

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39. MULTIPLE REPRESENTATIONS You are standing on a cliff above an ocean. You see a sailboat from your vantage point 30 feet above the ocean.

a. Drawing a Diagram Draw and label a diagram of the situation. **a–c. See margin.**

b. Making a Table Make a table showing the angle of depression and the length of your line of sight. Use the angles 40°, 50°, 60°, 70°, and 80°.

c. Drawing a Graph Graph the values you found in part (b), with the angle measures on the x-axis.

d. Making a Prediction Predict the length of the line of sight when the angle of depression is 30°. ***Sample answer:* 60 ft**

C **40. ALGEBRA** If $\triangle EQU$ is equilateral and $\triangle RGT$ is a right triangle with $RG = 2$, $RT = 1$, and $m\angle T = 90°$, show that $\sin E = \cos G$. **See margin.**

41. CHALLENGE Make a conjecture about the relationship between sine and cosine values.

a. Make a table that gives the sine and cosine values for the acute angles of a 45°-45°-90° triangle, a 30°-60°-90° triangle, a 34°-56°-90° triangle, and a 17°-73°-90° triangle. **See margin.**

b. Compare the sine and cosine values. What pattern(s) do you notice? **For complementary angles, the sine and cosine values are reversed, *i.e.* sin 30 = cos 60.**

c. Make a conjecture about the sine and cosine values in part (b). **If *A* and *B* are complementary, then sin *A* = cos *B*.**

d. Is the conjecture in part (c) true for right triangles that are not special right triangles? *Explain.* **Check students' work.**

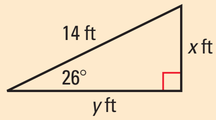
Using ALTERNATIVE METHODS

Another Way to Solve Example 5

MULTIPLE REPRESENTATIONS You can use the Pythagorean Theorem, tangent ratio, sine ratio, or cosine ratio to find the length of an unknown side of a right triangle. The decision of which method to use depends upon what information you have. In some cases, you can use more than one method to find the unknown length.

PROBLEM

SKATEBOARD RAMP You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and base of the ramp.



METHOD 1

Using a Cosine Ratio and the Pythagorean Theorem

STEP 1 **Find** the measure of the third angle.

$$26° + 90° + m\angle 3 = 180°$$ **Triangle Sum Theorem**

$$116° + m\angle 3 = 180°$$ **Combine like terms.**

$$m\angle 3 = 64°$$ **Subtract 116° from each side.**

STEP 2 **Use** the cosine ratio to find the height of the ramp.

$$\cos 64° = \frac{\text{adj.}}{\text{hyp.}}$$ **Write ratio for cosine of 64°.**

$$\cos 64° = \frac{x}{14}$$ **Substitute.**

$$14 \cdot \cos 64° = x$$ **Multiply each side by 14.**

$$6.1 \approx x$$ **Use a calculator to simplify.**

▸ The height is about 6.1 feet.

STEP 3 **Use** the Pythagorean Theorem to find the length of the base of the ramp.

$$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$$ **Pythagorean Theorem**

$$14^2 = 6.1^2 + y^2$$ **Substitute.**

$$196 = 37.21 + y^2$$ **Multiply.**

$$158.79 = y^2$$ **Subtract 37.21 from each side.**

$$12.6 \approx y$$ **Find the positive square root.**

▸ The length of the base is about 12.6 feet.

Alternative Strategy

Example 5 in previous lesson can be solved by using the Pythagorean Theorem instead of a trigonometric ratio. The third angle can also be found and used for the trigonometric ratios instead of the given angle. Students may feel more comfortable using the Pythagorean Theorem than a trigonometric ratio and may be less likely to make mistakes.

Avoiding Common Errors

Method 2 Encourage students to show all the steps when they solve $\tan 26° = \frac{6.1}{y}$: $y \cdot \tan 26° = 6.1$ and $y = \frac{6.1}{\tan 26°}$. That way they are less likely to write the incorrect statement that y is $\frac{\tan 26°}{6.1}$.

Graphing Calculator

Remind student that their calculator must be set on degrees, not radians, to get the correct values for sine, cosine, and tangent ratios.

4. If you have 2 of the 3 side lengths of a right triangle, then you would use the Pythagorean Theorem. If you have one angle and one side of the triangle, then you would use the trigonometric ratios.

5. The cosine ratio is the adjacent side over the hypotenuse, not opposite over adjacent; $\cos A = \frac{7}{25}$.

6a. About 56 ft; I drew a picture and saw that the height, angle, and distance form the tangent ratio, so I solved $\tan 75° = \frac{h}{15}$.

6b. *Sample answer:* You would need another triangle that would be similar to the given information, such as another tree that is in the same path with measurements to support the fact that the triangles are similar.

6c. You do not know the distance from the top of the tree to the end of the shadow, which forms the hypotenuse of the triangle.

METHOD 2 **Using a Tangent Ratio**

Use the tangent ratio and $h = 6.1$ feet to find the length of the base of the ramp.

$\tan 26° = \frac{\text{opp.}}{\text{adj.}}$	**Write ratio for tangent of 26°.**
$\tan 26° = \frac{6.1}{y}$	**Substitute.**
$y \cdot \tan 26° = 6.1$	**Multiply each side by *y*.**
$y = \frac{6.1}{\tan 26°}$	**Divide each side by tan 26°.**
$y \approx 12.5$	**Use a calculator to simplify.**

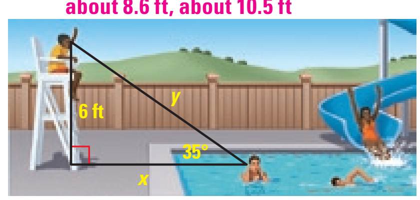
▶ The length of the base is about 12.5 feet.

Notice that when using the Pythagorean Theorem, the length of the base is 12.6 feet, but when using the tangent ratio, the length of the base is 12.5 feet. The tenth of a foot difference is due to the rounding error introduced when finding the height of the ramp and using that rounded value to calculate the length of the base.

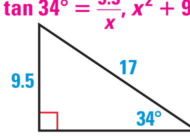
PRACTICE

1. **WHAT IF?** Suppose the length of the skateboard ramp is 20 feet. Find the height and base of the ramp. **about 8.8 ft, about 18 ft**

2. **SWIMMER** The angle of elevation from the swimmer to the lifeguard is 35°. Find the distance x from the swimmer to the base of the lifeguard chair. Find the distance y from the swimmer to the lifeguard. **about 8.6 ft, about 10.5 ft**

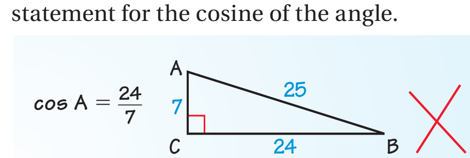


3. **ALGEBRA** Use the triangle below to write three different equations you can use to find the unknown leg length. ***Sample answer:*** **$\cos 34° = \frac{x}{17}$, $\tan 34° = \frac{9.5}{x}$, $x^2 + 9.5^2 = 17^2$**



4. **SHORT RESPONSE** *Describe* how you would decide whether to use the Pythagorean Theorem or trigonometric ratios to find the lengths of unknown sides of a right triangle. **See margin.**

5. **ERROR ANALYSIS** *Explain* why the student's statement is incorrect. Write a correct statement for the cosine of the angle. **See margin.**



6. **EXTENDED RESPONSE** You want to find the height of a tree in your yard. The tree's shadow is 15 feet long and you measure the angle of elevation from the end of the shadow to the top of tree to be 75°. **a–c. See margin.**
   a. Find the height of the tree. *Explain* the method you chose to solve the problem.
   b. What else would you need to know to solve this problem using similar triangles.
   c. *Explain* why you cannot use the sine ratio to find the height of the tree.

7.7 Solve Right Triangles

Before You used tangent, sine, and cosine ratios.

Now You will use inverse tangent, sine, and cosine ratios.

Why? So you can build a saddlerack, as in Ex. 39.

Key Vocabulary
- **solve a right triangle**
- **inverse tangent**
- **inverse sine**
- **inverse cosine**

To **solve a right triangle** means to find the measures of all of its sides and angles. You can solve a right triangle if you know either of the following:

- Two side lengths
- One side length and the measure of one acute angle

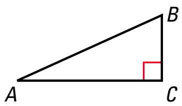
You have learned how to use the side lengths of a right triangle to find trigonometric ratios for the acute angles of the triangle. Once you know the tangent, the sine, or the cosine of an acute angle, you can use a calculator to find the measure of the angle.

READ VOCABULARY
The expression "$\tan^{-1} x$" is read as "the inverse tangent of x."

KEY CONCEPT *For Your Notebook*

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



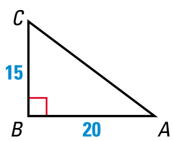
Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\qquad \tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\qquad \sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\qquad \cos^{-1} \frac{AC}{AB} = m\angle A$

EXAMPLE 1 Use an inverse tangent to find an angle measure

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Solution

Because $\tan A = \frac{15}{20} = \frac{3}{4} = 0.75$, $\tan^{-1} 0.75 = m\angle A$. Use a calculator.

$\tan^{-1} 0.75 \approx 36.86989765 \cdots$

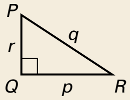
▶ So, the measure of $\angle A$ is approximately $36.9°$.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Use this diagram for Exercises 1–4.



1. If $PR = 12$ and $m\angle R = 19°$, find p. **11.3**

2. If $m\angle P = 58°$ and $r = 5$, find p. **8.0**

3. If $m\angle P = 60°$, and $p = 9$, find q. **10.4**

4. If $r = 8$ and $p = 12$, find q. **14.4**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
- See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

In a right triangle, how can you find all the sides and angles of the triangle? **Tell students they will learn how to answer this question by "solving" the triangle.**

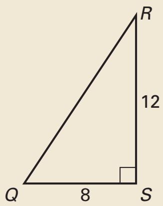
Motivating the Lesson

Suppose you are 5 feet tall and your shadow is 8 feet long. What is the angle of elevation of the sun? In this lesson students will learn how to find the measure of an acute angle of a right triangle if they know the lengths of two of the sides.

3 TEACH

Extra Example 1

Use a calculator to approximate the measure of $\angle Q$ to the nearest tenth of a degree. **56.3°**



Extra Example 2

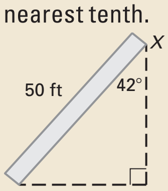
Let $\angle C$ be an acute angle in a right triangle. Use a calculator to approximate the measure of $\angle C$ to the nearest tenth of a degree.

a. $\sin C = 0.24$ **13.9°**

b. $\cos C = 0.37$ **68.3°**

Extra Example 3

Solve the right triangle formed by the water slide shown in the figure. Round decimal answers to the nearest tenth.



The angles are 42°, 48°, and 90°; the sides are 50 ft, about 33.5 ft, and about 37.2 ft.

EXAMPLE 2 Use an inverse sine and an inverse cosine

ANOTHER WAY
You can use the Table of Trigonometric Ratios on p. T7 to approximate $\sin^{-1} 0.87$ to the nearest degree. Find the number closest to 0.87 in the sine column and read the angle measure at the left.

Let $\angle A$ and $\angle B$ be acute angles in two right triangles. Use a calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree.

a. $\sin A = 0.87$ **b.** $\cos B = 0.15$

Solution

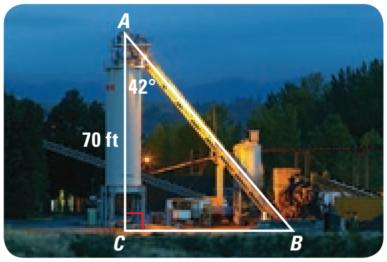
a. $m\angle A = \sin^{-1} 0.87 \approx 60.5°$ **b.** $m\angle B = \cos^{-1} 0.15 \approx 81.4°$

✓ GUIDED PRACTICE for Examples 1 and 2

1. Look back at Example 1. Use a calculator and an inverse tangent to approximate $m\angle C$ to the nearest tenth of a degree. **53.1°**
2. Find $m\angle D$ to the nearest tenth of a degree if $\sin D = 0.54$. **32.7°**

EXAMPLE 3 Solve a right triangle

Solve the right triangle. Round decimal answers to the nearest tenth.



Solution

STEP 1 **Find** $m\angle B$ by using the Triangle Sum Theorem.

$180° = 90° + 42° + m\angle B$

$48° = m\angle B$

STEP 2 **Approximate** BC by using a tangent ratio.

$\tan 42° = \frac{BC}{70}$ Write ratio for tangent of 42°.

$70 \cdot \tan 42° = BC$ Multiply each side by 70.

$70 \cdot 0.9004 \approx BC$ Approximate tan 42°.

$63 \approx BC$ Simplify and round answer.

ANOTHER WAY
You could also find *AB* by using the Pythagorean Theorem, or a sine ratio.

STEP 3 **Approximate** AB using a cosine ratio.

$\cos 42° = \frac{70}{AB}$ Write ratio for cosine of 42°.

$AB \cdot \cos 42° = 70$ Multiply each side by *AB*.

$AB = \frac{70}{\cos 42°}$ Divide each side by cos 42°.

$AB \approx \frac{70}{0.7431}$ Use a calculator to find cos 42°.

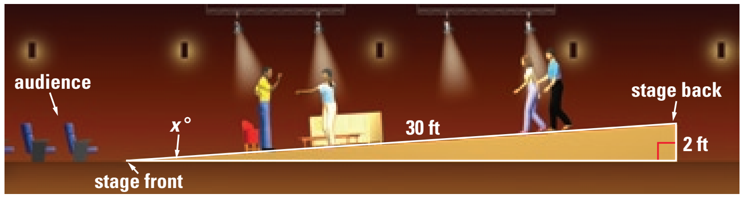
$AB \approx 94.2$ Simplify .

▶ The angle measures are 42°, 48°, and 90°. The side lengths are 70 feet, about 63 feet, and about 94 feet.

EXAMPLE 4 Solve a real-world problem

READ VOCABULARY
A *raked stage* slants upward from front to back to give the audience a better view.

THEATER DESIGN Suppose your school is building a *raked stage*. The stage will be 30 feet long from front to back, with a total rise of 2 feet. A rake (angle of elevation) of 5° or less is generally preferred for the safety and comfort of the actors. Is the raked stage you are building within the range suggested?



Solution

Use the sine and inverse sine ratios to find the degree measure x of the rake.

$$\sin x° = \frac{\text{opp.}}{\text{hyp.}} = \frac{2}{30} \approx 0.0667$$

$$x \approx \sin^{-1} 0.0667 \approx 3.824$$

▶ The rake is about 3.8°, so it is within the suggested range of 5° or less.

✓ **GUIDED PRACTICE** for Examples 3 and 4

3. Solve a right triangle that has a 40° angle and a 20 inch hypotenuse. **40°, 50°, and 90°, about 12.9 in., about 15.3 in. and 20 in.**

4. **WHAT IF?** In Example 4, suppose another raked stage is 20 feet long from front to back with a total rise of 2 feet. Is this raked stage safe? *Explain.* **No; the rake is 5.7° so it is slightly larger than the suggested range.**

7.7 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 13, and 35
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 29, 30, 35, 40, and 41
◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

SKILL PRACTICE

A

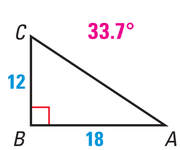
1. **VOCABULARY** Copy and complete: To solve a right triangle means to find the measures of all of its __?__ and __?__. **angles, sides**

2. ★ **WRITING** *Explain* when to use a trigonometric ratio to find a side length of a right triangle and when to use the Pythagorean Theorem. **See margin.**

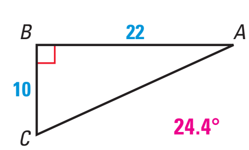
EXAMPLE 1
for Exs. 3–5

USING INVERSE TANGENTS Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

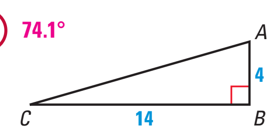
3. **33.7°**



4. **24.4°**



5. **74.1°**



Key Question to Ask for Example 3

- In Step 3, find *AB* by the Pythagorean Theorem. Do you get the same answer? *Explain.* **Both answers round to 94.2. They differ slightly because of the rounding error in the approximation of *BC*.**

Extra Example 4

A road rises 10 feet in a horizontal distance of 200 feet. What is the angle of inclination? **2.9°**

Vocabulary

The expression $\tan^{-1} x$ is a short way to indicate "the angle whose tangent ratio is x." "Be sure students understand that the raised "−1" is *not* an exponent, and that $\tan^{-1} x$ does not mean $\frac{1}{\tan x}$. Stress that this applies to $\sin^{-1} x$ and $\cos^{-1} x$ as well.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: In a right triangle, how can you find all the sides and angles of the triangle?

- **To "solve a triangle" means to find the measures of all the angles and all the sides.**
- **If you know the measures of a side length and an acute angle, you can use trig ratios to find the lengths of the other two sides.**
- **If you know the lengths of two sides, you can use an inverse trig ratio to find the measure of an angle and you can use the Pythagorean Theorem to find the length of the third side.**

You use the sine, cosine, and tangent ratios to find the length of a side of a right triangle. You can use the inverse sine, inverse cosine, or inverse tangent ratio to find the measures of the angles.

2. Use the Pythagorean Theorem if you have two sides of the triangle. Use a trigonometric ratio if you have an angle measure and a side length.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–9, 19–24, 37
Day 2:
Exs. 10–18, 34–36, 38, 39

Average:
Day 1:
Exs. 1, 2, 4, 5, 7–9, 19–28, 37
Day 2:
Exs. 12–16, 29–31, 34–36, 38–41

Advanced:
Day 1:
Exs. 1, 2, 4, 5, 8, 9, 19–28, 31, 32, 37
Day 2:
Exs. 14–18, 29, 30, 33*, 35, 36, 38–42*

Block:
Exs. 1, 2, 4, 5, 7–9, 12–16, 19–31, 34–41

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 12, 34, 36
Average: 4, 7, 14, 36, 37
Advanced: 4, 8, 16, 36, 38

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

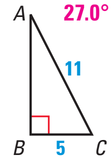
Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

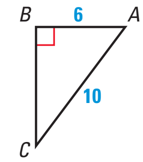
EXAMPLE 2 for Exs. 6–9

USING INVERSE SINES AND COSINES Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

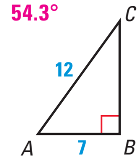
6. 27.0°



7. 53.1°



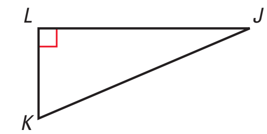
8. 54.3°



9. ★ **MULTIPLE CHOICE** Which expression is correct? B

Ⓐ $\sin^{-1}\frac{JL}{JK} = m\angle J$ Ⓑ $\tan^{-1}\frac{KL}{JL} = m\angle J$

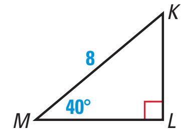
Ⓒ $\cos^{-1}\frac{JL}{JK} = m\angle K$ Ⓓ $\sin^{-1}\frac{JL}{KL} = m\angle K$



EXAMPLE 3 for Exs. 10–18

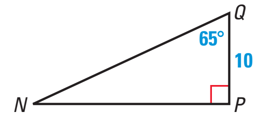
SOLVING RIGHT TRIANGLES Solve the right triangle. Round decimal answers to the nearest tenth.

10.



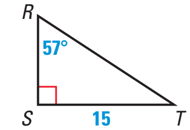
$K = 50°$, $KL \approx 5.1$, $ML \approx 6.1$

11.



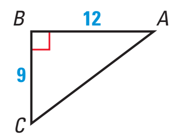
$N = 25°$, $NP \approx 21.4$, $NQ \approx 23.7$

12.



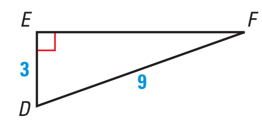
$T = 33°$, $RS \approx 9.7$, $RT \approx 17.9$

13.



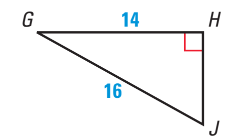
$A \approx 36.9°$, $B \approx 53.1°$, $AC = 15$

14.



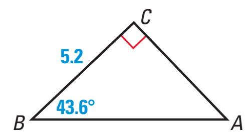
$D \approx 70.5°$, $F \approx 19.5°$, $EF \approx 8.5$

15.



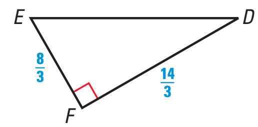
$G \approx 29°$, $J \approx 61°$, $HJ \approx 7.7$

16.



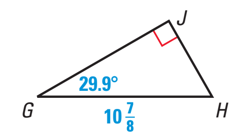
$A = 46.4°$, $AB \approx 7.2$, $AC \approx 5.0$

17.



$D \approx 29.7°$, $E \approx 60.3°$, $ED \approx 5.4$

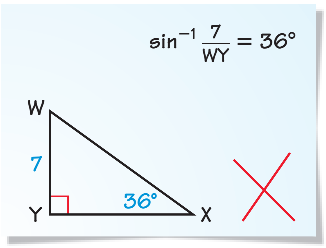
18.



$H = 60.1°$, $GJ \approx 9.4$, $JH \approx 5.4$

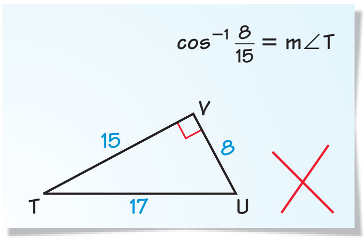
B **ERROR ANALYSIS** *Describe* and correct the student's error in using an inverse trigonometric ratio.

19.



19. *WX* should have been used instead of *WY*; $\sin^{-1}\frac{7}{WX} = 36°$.

20.



20. To determine the measure of angle *T* using cosine, the ratio is adjacent over hypotenuse; $\cos^{-1}\frac{15}{17} = T$.

CALCULATOR Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree.

21. $\sin A = 0.5$ 30° **22.** $\sin A = 0.75$ 48.6° **23.** $\cos A = 0.33$ 70.7° **24.** $\cos A = 0.64$ 50.2°

25. $\tan A = 1.0$ 45° **26.** $\tan A = 0.28$ 15.6° **27.** $\sin A = 0.19$ 11.0° **28.** $\cos A = 0.81$ 35.9°

○ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

29. ★ **MULTIPLE CHOICE** Which additional information would *not* be enough to solve $\triangle PRQ$? **B**

Ⓐ $m\angle P$ and PR Ⓑ $m\angle P$ and $m\angle R$

Ⓒ PQ and PR Ⓓ $m\angle P$ and PQ

30. ★ **WRITING** *Explain* why it is incorrect to say that $\tan^{-1} x = \frac{1}{\tan x}$. **$\tan^{-1}$ is the function which is used to determine the measure of an angle given the proper ratio of sides.**

C 31. **SPECIAL RIGHT TRIANGLES** If $\sin A = \frac{1}{2}\sqrt{2}$, what is $m\angle A$? If $\sin B = \frac{1}{2}\sqrt{3}$, what is $m\angle B$? **45°; 60°**

32. **TRIGONOMETRIC VALUES** Use the *Table of Trigonometric Ratios* on p. T7 to answer the questions.

a. What angles have nearly the same sine and tangent values? **0 to 10°**

b. What angle has the greatest difference in its sine and tangent value? **89°**

c. What angle has a tangent value that is double its sine value? **60°**

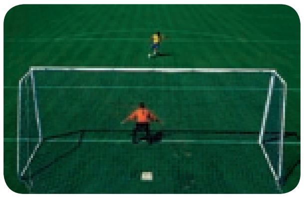
d. Is $\sin 2x$ equal to $2 \cdot \sin x$? **no**

33. **CHALLENGE** The perimeter of rectangle $ABCD$ is 16 centimeters, and the ratio of its width to its length is $1:3$. Segment BD divides the rectangle into two congruent triangles. Find the side lengths and angle measures of one of these triangles. **6 cm, 2cm, $2\sqrt{10}$ cm, 90°, about 18.4°, about 71.6°**

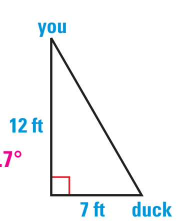
PROBLEM SOLVING

EXAMPLE 4 A
for Exs. 34–36

34. **SOCCER** A soccer ball is placed 10 feet away from the goal, which is 8 feet high. You kick the ball and it hits the crossbar along the top of the goal. What is the angle of elevation of your kick? **about 38.7°**

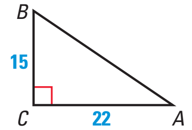


35. ★ **SHORT RESPONSE** You are standing on a footbridge in a city park that is 12 feet high above a pond. You look down and see a duck in the water 7 feet away from the footbridge. What is the angle of depression? *Explain* your reasoning. **about 59.7°; $90 - \tan^{-1}\frac{7}{12} \approx 59.7°$**



36. **CLAY** In order to unload clay easily, the body of a dump truck must be elevated to at least 55°. If the body of the dump truck is 14 feet long and has been raised 10 feet, will the clay pour out easily? **no**

37. **REASONING** For $\triangle ABC$ shown, each of the expressions $\sin^{-1}\frac{BC}{AB}$, $\cos^{-1}\frac{AC}{AB}$, and $\tan^{-1}\frac{BC}{AC}$ can be used to approximate the measure of $\angle A$. Which expression would you choose? *Explain* your choice.



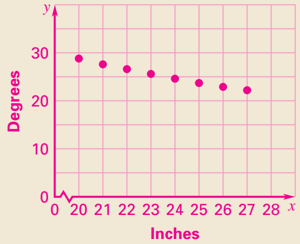
37. $\tan^{-1}\frac{BC}{AC}$; the information needed to determine the measure of A is given. If you used the tangent ratio, this will make the answer more accurate since no rounding has occurred.

Avoiding Common Errors

Exercise 41 Students may assume that for part (c) the wire will be half as long as the wire for part (b) since it only reaches halfway up the tower. Make sure they calculate both lengths. Review that the triangles are not similar so the ratio of their hypotenuses is not 1 : 2.

39a. See below.

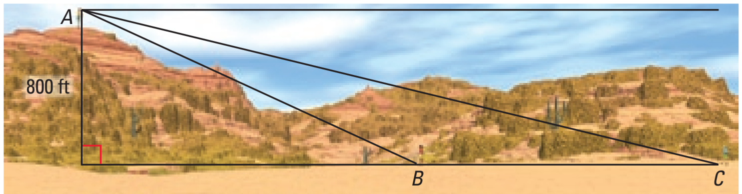
39b.



42. Statements (Reasons)

1. $\triangle ABC$ with altitude $\overline{CD}$ (Given)
2. $\sin A = \frac{CD}{b}$ and $\sin B = \frac{CD}{a}$ (Definition of sine)
3. $CD = b \sin A$ and $CD = a \sin B$ (Multiplication Property of Equality)
4. $b \sin A = a \sin B$ (Substitution Property of Equality)
5. $\frac{\sin A}{a} = \frac{\sin B}{b}$ (Division Property of Equality)

B **38. MULTI-STEP PROBLEM** You are standing on a plateau that is 800 feet above a basin where you can see two hikers.

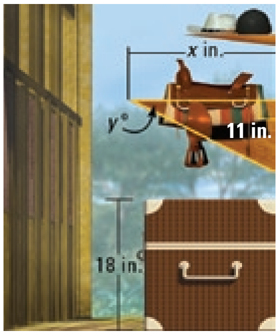


a. If the angle of depression from your line of sight to the hiker at B is 25°, how far is the hiker from the base of the plateau? **about 1716 ft**

b. If the angle of depression from your line of sight to the hiker at C is 15°, how far is the hiker from the base of the plateau? **about 2986 ft**

c. How far apart are the two hikers? *Explain.* **About 1270 ft; subtract the two distances to find out how far the two hikers are from each other.**

39. MULTIPLE REPRESENTATIONS A local ranch offers trail rides to the public. It has a variety of different sized saddles to meet the needs of horse and rider. You are going to build saddle racks that are 11 inches high. To save wood, you decide to make each rack fit each saddle.



a. **Making a Table** The lengths of the saddles range from 20 inches to 27 inches. Make a table showing the saddle rack length x and the measure of the adjacent angle $y°$. **a, b. See margin.**

b. **Drawing a Graph** Use your table to draw a scatterplot.

c. **Making a Conjecture** Make a conjecture about the relationship between the length of the rack and the angle needed. ***Sample answer:* The longer the rack, the closer to 20° the angle gets.**

40. ★ OPEN-ENDED MATH *Describe* a real-world problem you could solve using a trigonometric ratio.

40. ***Sample answer:*** **You want to know how tall your town's water tower is. You are standing 40 feet away from the base of the tower and the angle of elevation is 60°. How tall is the water tower?**

41. ★ EXTENDED RESPONSE Your town is building a wind generator to create electricity for your school. The builder wants your geometry class to make sure that the guy wires are placed so that the tower is secure. By safety guidelines, the distance along the ground from the tower to the guy wire's connection with the ground should be between 50% to 75% of the height of the guy wire's connection with the tower.

a. The tower is 64 feet tall. The builders plan to have the distance along the ground from the tower to the guy wire's connection with the ground be 60% of the height of the tower. How far apart are the tower and the ground connection of the wire? **38.4 ft**

b. How long will a guy wire need to be that is attached 60 feet above the ground? **about 71.2 ft**

c. How long will a guy wire need to be that is attached 30 feet above the ground? **about 48.7 ft**

d. Find the angle of elevation of each wire. Are the right triangles formed by the ground, tower, and wires *congruent*, *similar*, or *neither*? *Explain.*

e. *Explain* which trigonometric ratios you used to solve the problem. **I used tangent because the height and the distance along the ground form a tangent relationship for the angle of elevation.**

41d. **about 57.4°, about 38.0°; neither; the sides are not the same, so the triangles are not congruent, and the angles are not the same, so the triangles are not similar.**

○ = See WORKED-OUT SOLUTIONS in Student Resources ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

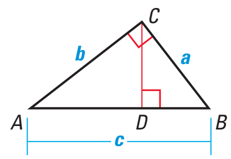
39a.

x (inches)	20	21	22	23	24	25	26	27
y (degrees)	28.8	27.6	26.6	25.6	24.6	23.7	22.9	22.2

C **42. CHALLENGE** Use the diagram of $\triangle ABC$.

GIVEN ▶ $\triangle ABC$ with altitude $\overline{CD}$.

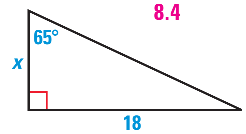
PROVE ▶ $\frac{\sin A}{a} = \frac{\sin B}{b}$ **See margin.**



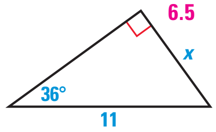
QUIZ

Find the value of x to the nearest tenth.

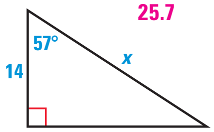
1. **8.4**



2. **6.5**

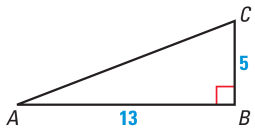


3. **25.7**



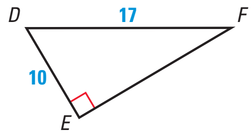
Solve the right triangle. Round decimal answers to the nearest tenth.

4.



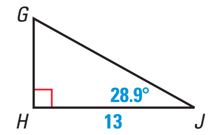
$A \approx 21.0°$, $C \approx 69.0°$, $AC \approx 13.9$

5.



$D \approx 54°$, $F \approx 36°$, $EF \approx 13.7$

6.



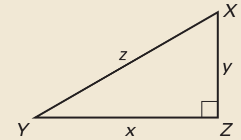
$G = 61.1°$, $GH \approx 7.2$, $JG \approx 14.8$

5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

Use this diagram for Exercises 1–3.



1. If $x = 9$ and $z = 11$, find $m\angle X$ to the nearest tenth of a degree. **54.9°**

2. If $y = 5$ and $z = 12$, find $m\angle X$ to the nearest tenth of a degree. **65.4°**

3. If $m\angle Y = 17.4°$ and $z = 12$, solve $\triangle XYZ$. **The angles are 17.4°, 72.6°, and 90°; the sides are 12, about 3.6, and about 11.5.**

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

Extension Law of Sines and Law of Cosines

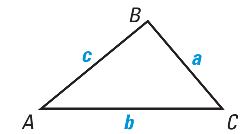
GOAL Use trigonometry with acute and obtuse triangles.

The trigonometric ratios you have seen so far in this chapter can be used to find angle and side measures in right triangles. You can use the Law of Sines to find angle and side measures in *any* triangle.

KEY CONCEPT *For Your Notebook*

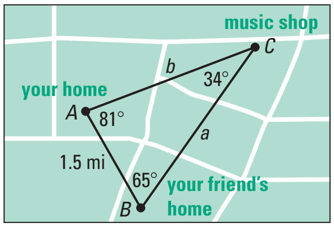
Law of Sines

If $\triangle ABC$ has sides of length a, b, and c as shown, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



EXAMPLE 1 Find a distance using Law of Sines

DISTANCE **Use the information in the diagram to determine how much closer you live to the music store than your friend does.**



Solution

STEP 1 **Use** the Law of Sines to find the distance a from your friend's home to the music store.

$\frac{\sin A}{a} = \frac{\sin C}{c}$ **Write Law of Sines.**

$\frac{\sin 81°}{a} = \frac{\sin 34°}{1.5}$ **Substitute.**

$a \approx 2.6$ **Solve for *a*.**

STEP 2 **Use** the Law of Sines to find the distance b from your home to the music store.

$\frac{\sin B}{b} = \frac{\sin C}{c}$ **Write Law of Sines.**

$\frac{\sin 65°}{b} = \frac{\sin 34°}{1.5}$ **Substitute.**

$b \approx 2.4$ **Solve for *b*.**

STEP 3 **Subtract** the distances.

$a - b \approx 2.6 - 2.4 = 0.2$

▸ You live about 0.2 miles closer to the music store.

1 PLAN AND PREPARE

Warm-Up Exercises

1. Solve $\frac{x}{5} = \frac{6}{3}$. **10**
2. If $\sin A = 0.8357$, what is $m\angle A$ to the nearest degree? **57°**
3. Solve $x^2 = 4^2 + 5^2 - 2(4)(5)(0.6)$. Write your answer as a radical and as a decimal to the nearest hundredth. $\pm\sqrt{17} \approx \pm 4.12$

2 FOCUS AND MOTIVATE

Essential Question

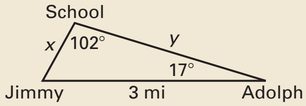
Big Idea 3

How do you find sides and angles in acute and obtuse triangles?
Tell students they will learn how to answer this question by using two properties called the Law of Sines and the Law of Cosines.

3 TEACH

Extra Example 1

Use the information in the diagram to find how much closer, to the nearest hundredth mile, Jimmy lives to school than Adolph does. **1.79 mi**



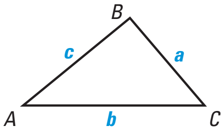
LAW OF COSINES You can also use the Law of Cosines to solve any triangle.

KEY CONCEPT *For Your Notebook*

Law of Cosines

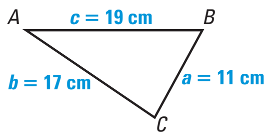
If $\triangle ABC$ has sides of length a, b, and c, then:

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$



EXAMPLE 2 Find an angle measure using Law of Cosines

In $\triangle ABC$ at the right, $a = 11$ cm, $b = 17$ cm, and $c = 19$ cm. Find $m\angle C$.



Solution

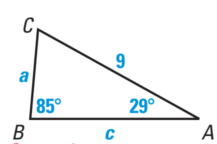
$c^2 = a^2 + b^2 - 2ab \cos C$	**Write Law of Cosines.**
$19^2 = 11^2 + 17^2 - 2(11)(17) \cos C$	**Substitute.**
$0.1310 = \cos C$	**Solve for cos *C*.**
$m\angle C \approx 82°$	**Find $\cos^{-1}(0.1310)$.**

PRACTICE

EXAMPLE 1 for Exs. 1–3

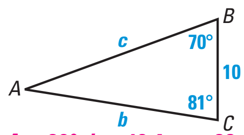
LAW OF SINES **Use the Law of Sines to solve the triangle. Round decimal answers to the nearest tenth.**

1.



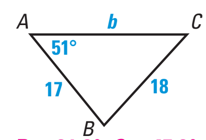
$C = 66°$, $a \approx 4.4$, $c \approx 8.3$

2.



$A = 29°$, $b \approx 19.4$, $c \approx 20.4$

3.

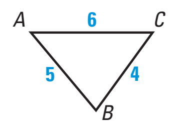


$B \approx 81.8°$, $C \approx 47.2°$, $b \approx 22.9$

EXAMPLE 2 for Exs. 4–7

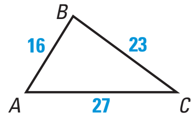
LAW OF COSINES **Use the Law of Cosines to solve the triangle. Round decimal answers to the nearest tenth.**

4.



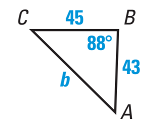
$B \approx 82.8°$, $C \approx 55.8°$, $A \approx 41.4°$

5.



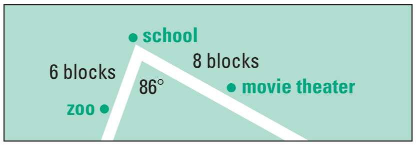
$A \approx 58.1°$, $B \approx 85.6°$, $C \approx 36.2$

6.



$A \approx 47.4°$, $C \approx 44.7°$, $b \approx 61.1$

7. DISTANCE Use the diagram at the right. Find the straight distance between the zoo and movie theater. about 10 blocks

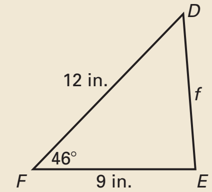


Study Strategy

The Law of Sines can be used to to find missing parts of a triangle when the given information can be represented as ASA, AAS, or SSA (there may be more than 1 solution for an SSA situation). The Law of Cosines can be used when you are given SAS or SSS.

Extra Example 2

In $\triangle DEF$, $d = 9$ in., $e = 12$ in., and $m\angle F = 46°$. Find f to the nearest hundredth. **8.66 in.**



Avoiding Common Errors

Example 2 When students see the equation $19^2 = 11^2 + 17^2 - 2(11)(17) \cos C$, some may subtract $2(11)(17)$ from $11^2 + 17^2$. Show them a simpler statement having the same structure, such as $a = b + c - dx$, and ask them what operations they would perform to solve for x.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you find sides or angles in acute and obtuse triangles?

- **The Law of Sines is the property that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.**
- **The Law of Cosines is the property that $a^2 = b^2 + c^2 - 2bc \cos A$.**

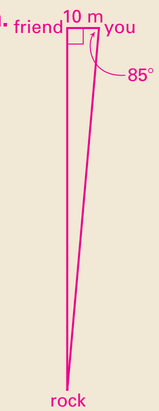
Use the Law of Sines or the Law of Cosines.

4 PRACTICE AND APPLY

Teaching Strategy

Exercise 5 When solving a triangle given three sides and no angles, students should start by using the Law of Cosines to find the largest angle, in case it is obtuse. Then they can use the Law of Sines to find another angle.

2a.



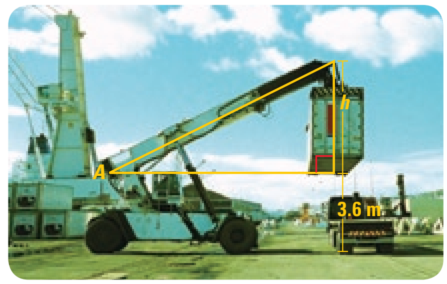
5a. $9\sqrt{3}$. *Sample answer:* I used the Pythagorean Theorem with $\triangle AEC$. Since $\triangle ACD$ is isosceles, and $\overline{AD}$ is bisected by $\overline{EC}$, $\overline{EC}$ is the altitude and is therefore perpendicular to the base.

5b. About 10.9°. *Sample answer:* I found the measure of $\angle ACE$ using the sine ratio, then I found the measure of $\angle ACB$ because it is supplementary to $\angle ACE$. The Law of Cosines allowed me to find AB and then I used the Law of Sines to find the measure of $\angle ABC$.

5c. *Sample answer:* Part (a): Since the hypotenuse is 18 and the given leg is half of that, the triangle must be a 30°-60°-90° triangle and therefore, the other leg must be $9\sqrt{3}$; part (b): EB must equal $3x$ and since $x = 9\sqrt{3}$, $EB = 27\sqrt{3}$. Now you can use the inverse tangent function to determine the measure of $\angle ABC$.

MIXED REVIEW *of Problem Solving*

1. **MULTI-STEP PROBLEM** A *reach stacker* is a vehicle used to lift objects and move them between ships and land.



   a. The vehicle's arm is 10.9 meters long. The maximum measure of $\angle A$ is 60°. What is the greatest height h the arm can reach if the vehicle is 3.6 meters tall? **about 13.0 m**

   b. The vehicle's arm can extend to be 16.4 meters long. What is the greatest height its extended arm can reach? **about 17.8 m**

   c. What is the difference between the two heights the arm can reach above the ground? **about 4.8 m**

2. **EXTENDED RESPONSE** You and a friend are standing the same distance from the edge of a canyon. Your friend looks directly across the canyon at a rock. You stand 10 meters from your friend and estimate the angle between your friend and the rock to be 85°.

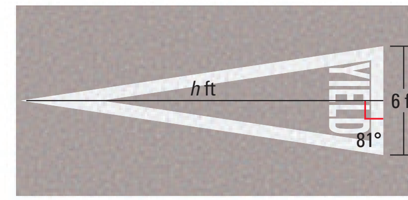
   a. Sketch the situation. **See margin.**

   b. *Explain* how to find the distance across the canyon. **Use the tangent ratio.**

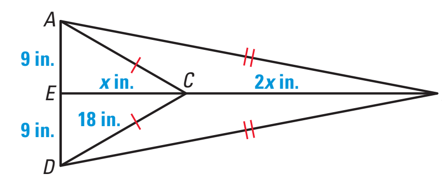
   c. Suppose the actual angle measure is 87°. How far off is your estimate of the distance? **about 76.5 m**

3. **SHORT RESPONSE** The international rules of basketball state the rim of the net should be 3.05 meters above the ground. If your line of sight to the rim is 34° and you are 1.7 meters tall, what is the distance from you to the rim? *Explain* your reasoning.
   about 2.4 m; $\sin 34° = \frac{1.35}{x}$

4. **GRIDDED ANSWER** The specifications for a *yield ahead* pavement marking are shown. Find the height h in feet of this isosceles triangle to the nearest tenth. **18.9**



5. **EXTENDED RESPONSE** Use the diagram to answer the questions. **a–c. See margin.**

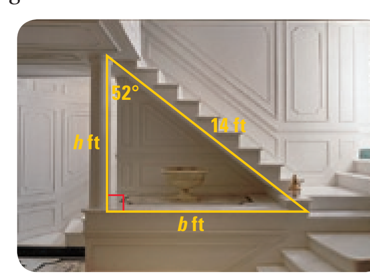


   a. Solve for x. *Explain* the method you chose.

   b. Find $m\angle ABC$. *Explain* the method you chose.

   c. *Explain* a different method for finding each of your answers in parts (a) and (b).

6. **SHORT RESPONSE** The triangle on the staircase below has a 52° angle and the distance along the stairs is 14 feet. What is the height h of the staircase? What is the length b of the base of the staircase?



   about 8.6 ft; about 11.0 ft

7. **GRIDDED ANSWER** The base of an isosceles triangle is 70 centimeters long. The altitude to the base is 75 centimeters long. Find the measure of a base angle to the nearest degree. **65**

7 CHAPTER SUMMARY

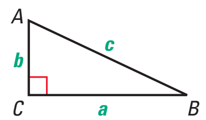
BIG IDEAS *For Your Notebook*

Big Idea 1

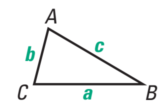
Using the Pythagorean Theorem and Its Converse

The Pythagorean Theorem states that in a right triangle the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b, so that $c^2 = a^2 + b^2$.

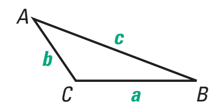
The Converse of the Pythagorean Theorem can be used to determine if a triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.



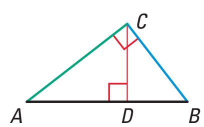
If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.

Big Idea 2

Using Special Relationships in Right Triangles

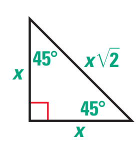
GEOMETRIC MEAN In right $\triangle ABC$, altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.

Also, $\frac{BD}{CD} = \frac{CD}{AD}$, $\frac{AB}{CB} = \frac{CB}{DB}$, and $\frac{AB}{AC} = \frac{AC}{AD}$.



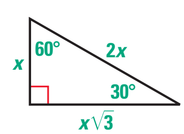
SPECIAL RIGHT TRIANGLES

45°-45°-90° Triangle



hypotenuse = leg $\cdot \sqrt{2}$

30°-60°-90° Triangle



hypotenuse = 2 $\cdot$ shorter leg

longer leg = shorter leg $\cdot \sqrt{3}$

Big Idea 3

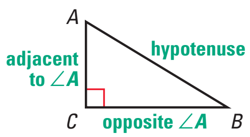
Using Trigonometric Ratios to Solve Right Triangles

The tangent, sine, and cosine ratios can be used to find unknown side lengths and angle measures of right triangles. The values of tan $x°$, sin $x°$, and cos $x°$ depend only on the angle measure and not on the side length.

$$\tan A = \frac{\text{opp.}}{\text{adj.}} = \frac{BC}{AC} \qquad \tan^{-1}\frac{BC}{AC} = m\angle A$$

$$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{AB} \qquad \sin^{-1}\frac{BC}{AB} = m\angle A$$

$$\cos A = \frac{\text{adj.}}{\text{hyp.}} = \frac{AC}{AB} \qquad \cos^{-1}\frac{AC}{AB} = m\angle A$$



Additional Resources

The following resources are available to help review the materials in this chapter.

Chapter Resource Book

- Chapter Review Games and Activities
- Cumulative Practice

Student Resources in Spanish

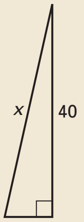
@HomeTutor

Vocabulary Practice

Vocabulary practice is available at **my.hrw.com**

Extra Example 1

Find the value of x. **41**



7 CHAPTER REVIEW

@HomeTutor
my.hrw.com
- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see p. PT2.

- Pythagorean triple
- trigonometric ratio
- tangent
- sine
- cosine
- angle of elevation
- angle of depression
- solve a right triangle
- inverse tangent
- inverse sine
- inverse cosine

VOCABULARY EXERCISES

1. Copy and complete: A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation _?_. $a^2 + b^2 = c^2$

2. **WRITING** What does it mean to solve a right triangle? What do you need to know to solve a right triangle? **To find the measure of all three sides and all three angles; 2 side lengths, or 1 side length and 1 acute angle.**

3. **WRITING** *Describe* the difference between an angle of depression and an angle of elevation. ***Sample answer:* The difference is your perspective on the situation. The angle of depression is the measure from your line of sight down, and the angle of elevation is the measure from your line of sight up, but if you construct the parallel lines in any situation, the angles are alternate interior angles and are congruent by the Alternate Interior Angles Theorem.**

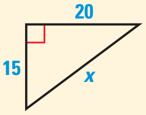
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of this chapter.

7.1 Apply the Pythagorean Theorem

EXAMPLE

Find the value of x.



Because x is the length of the hypotenuse of a right triangle, you can use the Pythagorean Theorem to find its value.

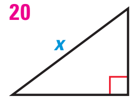
$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$		**Pythagorean Theorem**
$x^2 = 15^2 + 20^2$		**Substitute.**
$x^2 = 625$		**Simplify.**
$x = 25$		**Find the positive square root.**

EXERCISES

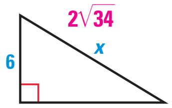
EXAMPLES 1 and 2 for Exs. 4–6

Find the unknown side length x.

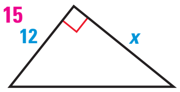
4. **20**



5. $2\sqrt{34}$



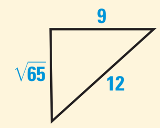
6. **15**



7.2 Use the Converse of the Pythagorean Theorem

EXAMPLE

Tell whether the given triangle is a right triangle.



Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

$12^2 \stackrel{?}{=} (\sqrt{65})^2 + 9^2$

$144 \stackrel{?}{=} 65 + 81$

$144 < 146$

The triangle is not a right triangle. It is an acute triangle.

EXERCISES

EXAMPLE 2
for Exs. 7–12

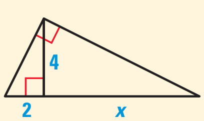
Classify the triangle formed by the side lengths as *acute*, *right*, or *obtuse*.

7. 6, 8, 9 acute
8. 4, 2, 5 obtuse
9. 10, $2\sqrt{2}$, $6\sqrt{3}$ right
10. 15, 20, 15 acute
11. 3, 3, $3\sqrt{2}$ right
12. 13, 18, $3\sqrt{55}$ obtuse

7.3 Use Similar Right Triangles

EXAMPLE

Find the value of *x*.



By using the Geometric Mean (Altitude) Theorem, you know that 4 is the geometric mean of x and 2.

$\frac{x}{4} = \frac{4}{2}$ **Write a proportion.**

$2x = 16$ **Cross Products Property**

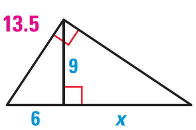
$x = 8$ **Divide.**

EXERCISES

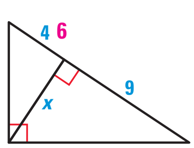
EXAMPLES 2 and 3
for Exs. 13–18

Find the value of *x*.

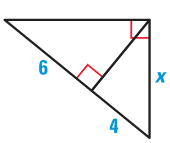
13. 13.5



14.

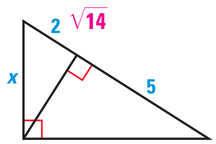


15.

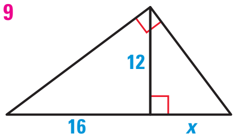


$2\sqrt{10}$

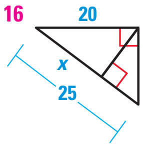
16.



17. 9

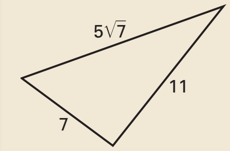


18. 16



Extra Example 2

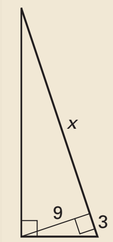
Tell whether the given triangle is a right triangle.



No, the triangle is obtuse.

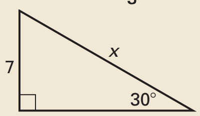
Extra Example 3

Find the value of *x*. 27



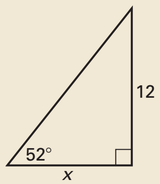
Extra Example 4

Find the length of the hypotenuse. 14



Extra Example 5

Find the value of x. 9.38

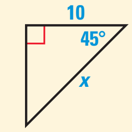


7 CHAPTER REVIEW

7.4 Special Right Triangles

EXAMPLE

Find the length of the hypotenuse.



By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • $\sqrt{2}$ **45°-45°-90° Triangle Theorem**

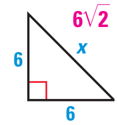
$x = 10\sqrt{2}$ **Substitute.**

EXAMPLES 1, 2, and 5 for Exs. 19–21

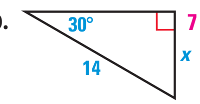
EXERCISES

Find the value of x. Write your answer in simplest radical form.

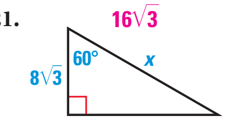
19. $6\sqrt{2}$



20. 7



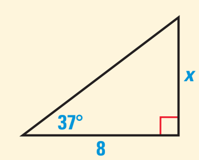
21. $16\sqrt{3}$



7.5 Apply the Tangent Ratio

EXAMPLE

Find the value of x.



$\tan 37° = \frac{\text{opp.}}{\text{adj.}}$ **Write ratio for tangent of 37°.**

$\tan 37° = \frac{x}{8}$ **Substitute.**

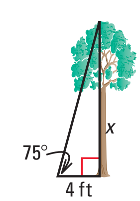
$8 \cdot \tan 37° = x$ **Multiply each side by 8.**

$6 \approx x$ **Use a calculator to simplify.**

EXAMPLE 2 for Exs. 22–26

EXERCISES

In Exercises 22 and 23, use the diagram.

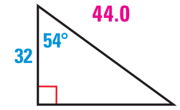


22. The angle between the bottom of a fence and the top of a tree is 75°. The tree is 4 feet from the fence. How tall is the tree? Round your answer to the nearest foot. 15 ft

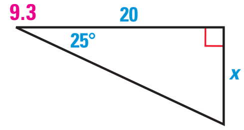
23. In Exercise 22, how tall is the tree if the angle is 55°? about 5.7 ft

Find the value of x to the nearest tenth.

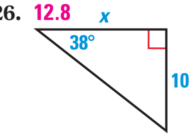
24. 44.0



25. 9.3



26. 12.8



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Chapter Review Practice

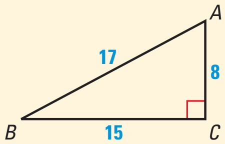
7.6 Apply the Sine and Cosine Ratios

EXAMPLE

Find sin A and sin B.

$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{BC}{BA} = \frac{15}{17} \approx 0.8824$

$\sin B = \frac{\text{opp.}}{\text{hyp.}} = \frac{AC}{AB} = \frac{8}{17} \approx 0.4706$

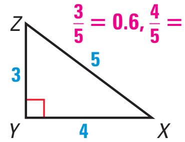


EXERCISES

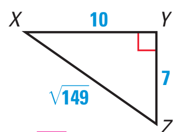
EXAMPLES 1 and 2 for Exs. 27–29

Find sin X and cos X. Write each answer as a fraction, and as a decimal. Round to four decimals places, if necessary.

27. $\frac{3}{5} = 0.6, \frac{4}{5} = 0.8$

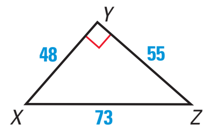


28.



$\frac{7\sqrt{149}}{149} \approx 0.5735, \frac{10\sqrt{149}}{149} \approx 0.8192$

29.



$\frac{55}{73} \approx 0.7534, \frac{48}{73} \approx 0.6575$

7.7 Solve Right Triangles

EXAMPLE

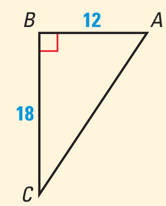
Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

Because $\tan A = \frac{18}{12} = \frac{3}{2} = 1.5$, $\tan^{-1} 1.5 = m\angle A$.

Use a calculator to evaluate this expression.

$\tan^{-1} 1.5 \approx 56.3099324 \ldots$

So, the measure of $\angle A$ is approximately 56.3°.

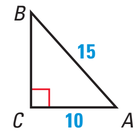


EXERCISES

EXAMPLE 3 for Exs. 30–33

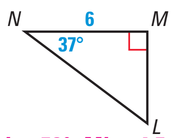
Solve the right triangle. Round decimal answers to the nearest tenth.

30.



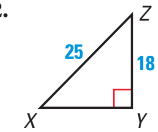
$A \approx 48.2°$, $B \approx 41.8°$, $BC \approx 11.2$

31.



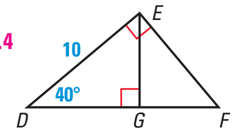
$L = 53°$, $ML \approx 4.5$, $NL \approx 7.5$

32.



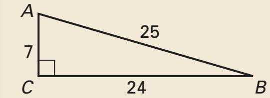
$X \approx 46.1°$, $Z \approx 43.9°$, $XY \approx 17.3$

33. Find the measures of $\angle GED$, $\angle GEF$, and $\angle EFG$. Find the lengths of $\overline{EG}$, $\overline{DF}$, $\overline{EF}$. 50°, 40°, 50°; about 6.4, about 13.1, about 8.4



Extra Example 6

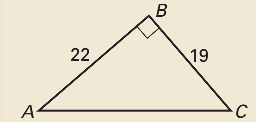
Find cos A and cos B.



$\cos A = \frac{7}{25} = 0.28$; $\cos B = \frac{24}{25} = 0.96$

Extra Example 7

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree. 40.8°



Additional Resources

Assessment Book

- Chapter Test, Levels A, B, C
- Standardized Chapter Test
- SAT/ACT Chapter Test
- Alternative Assessment

ExamView™ Assessment Suite

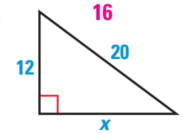
Chapter Test

Easily-readable reduced copies of Chapter Test B, the Standardized Chapter Test, and the Alternative Assessment from the Assessment Book can be found at the beginning of this chapter.

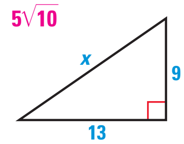
7 CHAPTER TEST

Find the value of x. Write your answer in simplest radical form.

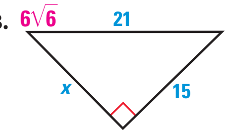
1. 16



2. $5\sqrt{10}$



3. $6\sqrt{6}$

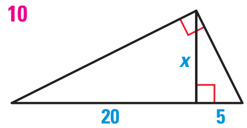


Classify the triangle as *acute*, *right*, or *obtuse*.

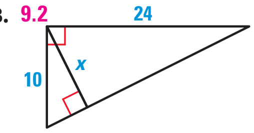
4. 5, 15, $5\sqrt{10}$ right
5. 4.3, 6.7, 8.2 obtuse
6. 5, 7, 8 acute

Find the value of x. Round decimal answers to the nearest tenth.

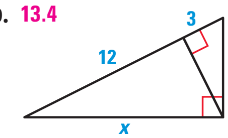
7. 10



8. 9.2

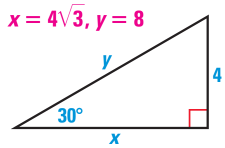


9. 13.4

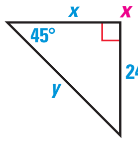


Find the value of each variable. Write your answer in simplest radical form.

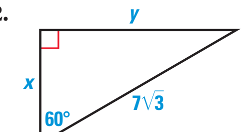
10. $x = 4\sqrt{3}$, $y = 8$



11. $x = 24$, $y = 24\sqrt{2}$



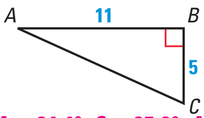
12.



$x = \frac{7\sqrt{3}}{2}$, $y = \frac{21}{2}$

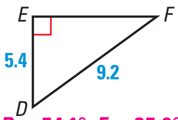
Solve the right triangle. Round decimal answers to the nearest tenth.

13.



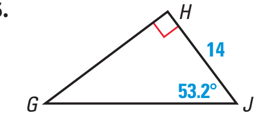
$A \approx 24.4°$, $C \approx 65.6°$, $AC \approx 12.1$

14.



$D \approx 54.1°$, $F \approx 35.9°$, $EF \approx 7.4$

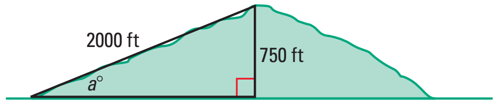
15.



$G = 36.8°$, $GH \approx 18.7$, $GJ \approx 23.4$

16. **FLAGPOLE** Julie is 6 feet tall. If she stands 15 feet from the flagpole and holds a cardboard square, the edges of the square line up with the top and bottom of the flagpole. Approximate the height of the flagpole. about 43.5 ft

17. **HILLS** The length of a hill in your neighborhood is 2000 feet. The height of the hill is 750 feet. What is the angle of elevation of the hill? about 22°



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GRAPH AND SOLVE QUADRATIC EQUATIONS

The graph of $y = ax^2 + bx + c$ is a parabola that opens upward if $a > 0$ and opens downward if $a < 0$. The x-coordinate of the vertex is $-\frac{b}{2a}$. The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

EXAMPLE 1 Graph a quadratic function

Graph the equation $y = -x^2 + 4x - 3$.

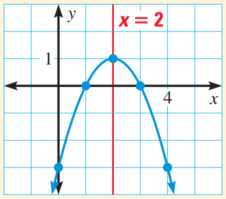
Because $a = -1$ and $-1 < 0$, the graph opens downward.

The vertex has x-coordinate $-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$.

The y-coordinate of the vertex is $-(2)^2 + 4(2) - 3 = 1$.

So, the vertex is (2, 1) and the axis of symmetry is $x = 2$.

Use a table of values to draw a parabola through the plotted points.



EXAMPLE 2 Solve a quadratic equation by graphing

Solve the equation $x^2 - 2x = 3$.

Write the equation in the standard form $ax^2 + bx + c = 0$:

$x^2 - 2x - 3 = 0$.

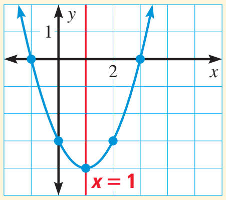
Graph the related quadratic function $y = x^2 - 2x - 3$, as shown.

The x-intercepts of the graph are -1 and 3.

So, the solutions of $x^2 - 2x = 3$ are -1 and 3.

Check the solution algebraically.

$(-1)^2 - 2(-1) \stackrel{?}{=} 3 \rightarrow 1 + 2 = 3$ $\qquad$ $(3)^2 - 2(3) \stackrel{?}{=} 3 \rightarrow 9 - 6 = 3$ ✓



EXERCISES

EXAMPLE 1 for Exs. 1–6

Graph the quadratic function. Label the vertex and axis of symmetry. 1–6. See margin.

1. $y = x^2 - 6x + 8$
2. $y = -x^2 - 4x + 2$
3. $y = 2x^2 - x - 1$
4. $y = 3x^2 - 9x + 2$
5. $y = \frac{1}{2}x^2 - x + 3$
6. $y = -4x^2 + 6x - 5$

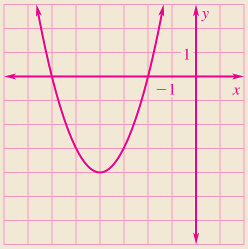
EXAMPLE 2 for Exs. 7–18

Solve the quadratic equation by graphing. Check solutions algebraically.

7. $x^2 = x + 6$ $-2, 3$
8. $4x + 4 = -x^2$ -2
9. $2x^2 = -8$ no solution
10. $3x^2 + 2 = 14$ $-2, 2$
11. $-x^2 + 4x - 5 = 0$ no solution
12. $2x - x^2 = -15$ $-3, 5$
13. $\frac{1}{4}x^2 = 2x$ $0, 8$
14. $x^2 + 3x = 4$ $-4, 1$
15. $x^2 + 8 = 6x$ $2, 4$
16. $x^2 = 9x - 1$ about 0.113, about 8.89
17. $-25 = x^2 + 10x$ -5
18. $x^2 + 6x = 0$ $-6, 0$

Extra Example 1

Graph the equation $y = x^2 + 8x + 12$.

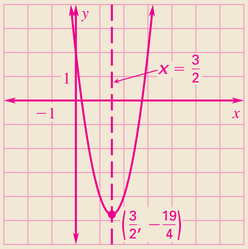


Extra Example 2

Solve the equation $x^2 + 2x = 8$.

$-4, 2$

4.



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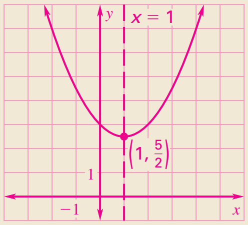

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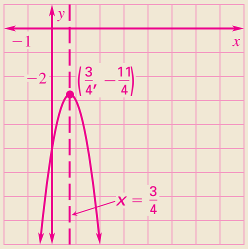
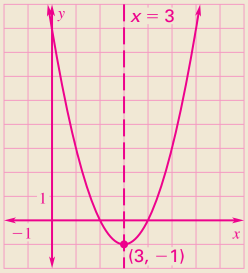


1.



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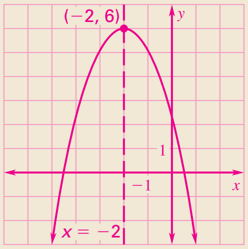

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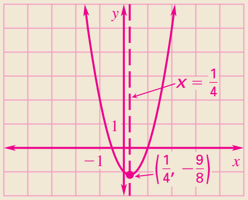


Test-Taking Strategy

Eliminate as many incorrect answer choices as you can. If you do not know how to solve the problem directly, you can sometimes use common sense with numbers to eliminate all but one of the choices.

Reading Strategy

Problem 1 Notice that the sides of $\triangle JKL$ are labeled in miles, but the question asks for the time in hours. Encourage students to read the problem carefully and find what is given and what is to be found.

Reading Strategy

Problem 2 Students should understand that an altitude of a triangle goes from a vertex to the opposite side and is perpendicular to that side. It is not equal to a side of the triangle unless the triangle is a right triangle. Also, students should notice that the triangle is equilateral, so any altitude will form two 30°-60°-90° triangles.

7 ★ Standardized TEST PREPARATION

MULTIPLE CHOICE QUESTIONS

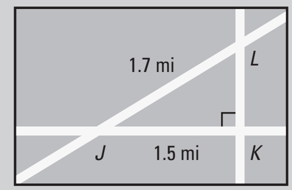
If you have difficulty solving a multiple choice question directly, you may be able to use another approach to eliminate incorrect answer choices and obtain the correct answer.

PROBLEM 1

You ride your bike at an average speed of 10 miles per hour. How long does it take you to ride one time around the triangular park shown in the diagram?

Ⓐ 0.1 h Ⓑ 0.2 h

Ⓒ 0.3 h Ⓓ 0.4 h



METHOD 1

SOLVE DIRECTLY The park is a right triangle. Use the Pythagorean Theorem to find KL. Find the perimeter of $\triangle JKL$. Then find how long it takes to ride around the park.

STEP 1 **Find** KL. Use the Pythagorean Theorem.

$$JK^2 + KL^2 = JL^2$$
$$1.5^2 + KL^2 = 1.7^2$$
$$2.25 + KL^2 = 2.89$$
$$KL^2 = 0.64$$
$$KL = 0.8$$

STEP 2 **Find** the perimeter of $\triangle JKL$.

$$P = JK + JL + KL$$
$$= 1.5 + 1.7 + 0.8$$
$$= 4 \text{ mi}$$

STEP 3 **Find** the time t (in hours) it takes you to go around the park.

$$\text{Rate} \times \text{Time} = \text{Distance}$$
$$(10 \text{ mi/h}) \cdot t = 4 \text{ mi}$$
$$t = 0.4 \text{ h}$$

The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

METHOD 2

ELIMINATE CHOICES Another method is to find how far you can travel in the given times to eliminate choices that are not reasonable.

STEP 1 **Find** how far you will travel in each of the given times. Use the formula $rt = d$.

Choice A: $0.1(10) = 1$ mi

Choice B: $0.2(10) = 2$ mi

Choice C: $0.3(10) = 3$ mi

Choice D: $0.4(10) = 4$ mi

The distance around two sides of the park is $1.5 + 1.7 = 3.2$ mi. But you need to travel around all three sides, which is longer.

Since $1 < 3.2$, $2 < 3.2$, and $3 < 3.2$. You can eliminate choices A, B, and C.

STEP 2 **Check** that D is the correct answer. If the distance around the park is 4 miles, then

$$KL = 4 - JK - JL$$
$$= 4 - 1.5 - 1.7 = 0.8 \text{ mi.}$$

Apply the Converse of the Pythagorean Theorem.

$$0.8^2 + 1.5^2 \stackrel{?}{=} 1.7^2$$
$$0.64 + 2.25 \stackrel{?}{=} 2.89$$
$$2.89 = 2.89 \checkmark$$

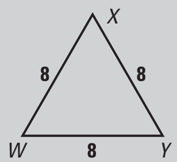
The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

PROBLEM 2

What is the height of $\triangle WXY$?

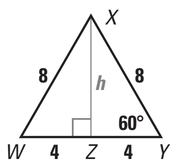
(A) 4 (B) $4\sqrt{3}$

(C) 8 (D) $8\sqrt{3}$



METHOD 1

SOLVE DIRECTLY Draw altitude $\overline{XZ}$ to form two congruent 30°-60°-90° triangles.



Let h be the length of the longer leg of $\triangle XZY$. The length of the shorter leg is 4.

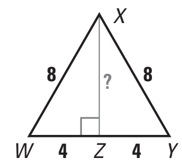
longer leg $= \sqrt{3} \cdot$ shorter leg

$h = 4\sqrt{3}$

The correct answer is B. (A) (B) (C) (D)

METHOD 2

ELIMINATE CHOICES Another method is to use theorems about triangles to eliminate incorrect choices. Draw altitude $\overline{XZ}$ to form two congruent right triangles.



Consider $\triangle XZW$. By the Triangle Inequality Theorem, $XW < WZ + XZ$. So, $8 < 4 + XZ$ and $XZ > 4$. You can eliminate choice A. Also, XZ must be less than the hypotenuse of $\triangle XWZ$. You can eliminate choices C and D.

The correct answer is B. (A) (B) (C) (D)

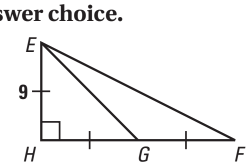
PRACTICE

Explain **why you can eliminate the highlighted answer choice.**

1. In the figure shown, what is the length of $\overline{EF}$?

(A) 9 (B) $\cancel{9\sqrt{2}}$

(C) 18 (D) $9\sqrt{5}$



2. Which of the following lengths are side lengths of a right triangle?

(A) ~~2, 21, 23~~ (B) 3, 4, 5 (C) 9, 16, 18 (D) 11, 16, 61

3. In $\triangle PQR$, $PQ = QR = 13$ and $PR = 10$. What is the length of the altitude drawn from vertex Q?

(A) 10 (B) 11 (C) 12 (D) ~~13~~

Answers

1. $\overline{EF}$ must be longer than 18 for the points to form a triangle.

2. 2 + 21 = 23 and in order for the sides to form a triangle the sum of the lengths of any two sides must be greater than the length of the third side. So these lengths do not form a triangle.

3. The altitude must be less than the hypotenuse, which is 13.

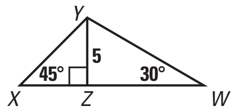
Answers

1. B
2. C
3. D
4. A
5. C
6. A
7. D

7 ★ Standardized TEST PRACTICE

MULTIPLE CHOICE

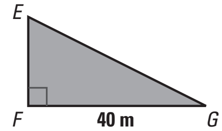
1. Which expression gives the correct length for XW in the diagram below?



Ⓐ $5 + 5\sqrt{2}$ Ⓑ $5 + 5\sqrt{3}$

Ⓒ $5\sqrt{3} + 5\sqrt{2}$ Ⓓ $5 + 10$

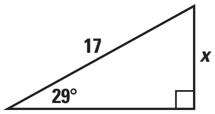
2. The area of $\triangle EFG$ is 400 square meters. To the nearest tenth of a meter, what is the length of side $\overline{EG}$?



Ⓐ 10.0 meters Ⓑ 20.0 meters

Ⓒ 44.7 meters Ⓓ 56.7 meters

3. Which expression can be used to find the value of x in the diagram below?



Ⓐ $\tan 29° = \frac{x}{17}$ Ⓑ $\cos 29° = \frac{x}{17}$

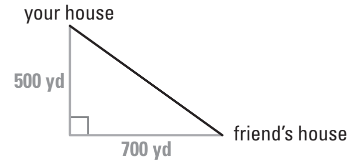
Ⓒ $\tan 61° = \frac{x}{17}$ Ⓓ $\cos 61° = \frac{x}{17}$

4. A fire station, a police station, and a hospital are not positioned in a straight line. The distance from the police station to the fire station is 4 miles. The distance from the fire station to the hospital is 3 miles. Which of the following could *not* be the distance from the police station to the hospital?

Ⓐ 1 mile Ⓑ 2 miles

Ⓒ 5 miles Ⓓ 6 miles

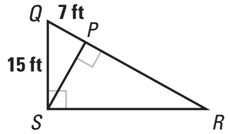
5. It takes 14 minutes to walk from your house to your friend's house on the path shown in gray. If you walk at the same speed, about how many minutes will it take on the path shown in black?



Ⓐ 6 minutes Ⓑ 8 minutes

Ⓒ 10 minutes Ⓓ 13 minutes

6. Which equation can be used to find QR in the diagram below?



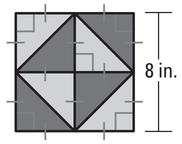
Ⓐ $\frac{QR}{15} = \frac{15}{7}$

Ⓑ $\frac{15}{QR} = \frac{QR}{8}$

Ⓒ $QR = \sqrt{15^2 + 27^2}$

Ⓓ $\frac{QR}{7} = \frac{7}{15}$

7. Stitches are sewn along the black line segments in the potholder shown below. There are 10 stitches per inch. Which is the closest estimate of the number of stitches used?

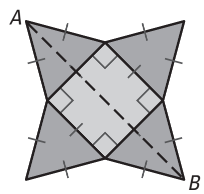


Ⓐ 480 Ⓑ 550

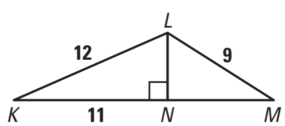
Ⓒ 656 Ⓓ 700

GRIDDED ANSWER

8. A design on a T-shirt is made of a square and four equilateral triangles. The side length of the square is 4 inches. Find the distance (in inches) from point A to point B. Round to the nearest tenth.

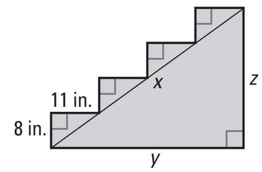


9. Use the diagram below. Find KM to the nearest tenth of a unit.

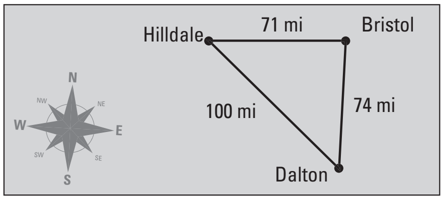


SHORT RESPONSE

10. The diagram shows the side of a set of stairs. In the diagram, the smaller right triangles are congruent. *Explain* how to find the lengths x, y, and z.



11. You drive due north from Dalton to Bristol. Next, you drive from Bristol to Hilldale. Finally, you drive from Hilldale to Dalton. Is Hilldale due west of Bristol? *Explain.*

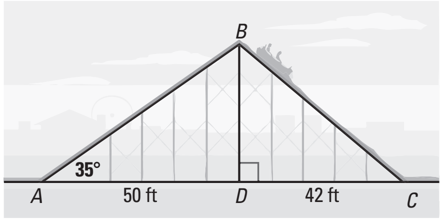


EXTENDED RESPONSE

12. The design for part of a water ride at an amusement park is shown. The ride carries people up a track along ramp $\overline{AB}$. Then riders travel down a water chute along ramp $\overline{BC}$.

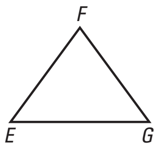
a. How high is the ride above point D? *Explain.*

b. What is the total distance from point A to point B to point C? *Explain.*



13. A formula for the area A of a triangle is *Heron's Formula*. For a triangle with side lengths EF, FG, and EG, the formula is

$A = \sqrt{s(s - EF)(s - FG)(s - EG)}$, where $s = \frac{1}{2}(EF + FG + EG)$.



a. In $\triangle EFG$ shown, $EF = FG = 15$, and $EG = 18$. Use Heron's formula to find the area of $\triangle EFG$.

b. Use the formula $A = \frac{1}{2}bh$ to find the area of $\triangle EFG$.

c. Use Heron's formula to *justify* that the area of an equilateral triangle with side length x is $A = \frac{x^2}{4}\sqrt{3}$.

8. 10.9

9. 18.6

10. Use the Pythagorean Theorem to find the length of the hypotenuse of the smaller triangle and since there are 4 such triangles, *x* is 4 times one hypotenuse. *y* is equivalent to 4 times the length of one small triangle and *z* is equivalent to 4 times the height of one small triangle.

11. No. *Sample answer:* If you use the law of cosines to determine the angle, you get about 87°. Since this is not 90°, Hilldale is not due west of Bristol.

12a. About 35 ft; the tangent ratio is used here to find the length of the side opposite the 35° angle, which represents the height of the triangle.

12b. About 116 ft; use the Pythagorean Theorem with *AD* and *DB* to find *AB*, and with *CD* and *DB* to find *BC*, and then find the sum of the distances.

13a. 108

13b. 108

13c. $x = \frac{1}{2}(x + x + x) = \frac{3}{2}x$, **so**

$A = \sqrt{\frac{3}{2}x\left(\frac{3}{2}x - x\right)\left(\frac{3}{2}x - x\right)\left(\frac{3}{2}x - x\right)}$

$= \sqrt{\frac{3}{2}x\left(\frac{1}{2}x\right)\left(\frac{1}{2}x\right)\left(\frac{1}{2}x\right)}$

$= \sqrt{\frac{3}{16}x^4}$

$= \frac{x^2}{4}\sqrt{3}$